Relative Positional Precision Explained in Everyday Language

Todd Horton, PE, PLS
January 2018
Todd Horton, PE, PLS, is an associate professor and the director of the land surveying and construction management degree programs at Parkland College in Champaign, Illinois, where he has taught since 1998. He also teaches surveying to University of Illinois civil engineering students. Beyond the campus, he presents continuing education seminars across the country for land surveyors and engineers.

Mr. Horton has several years of experience in planning, surveying, design, construction, and maintenance of civil engineering projects, including commercial structures, residential subdivisions, airfields, utility systems and highways. His previous employers include the US Air Force, the Illinois Department of Transportation, and engineering and surveying firms in central Illinois.

Mr. Horton is the faculty advisor of the Parkland College Student Chapter of the Illinois Professional Land Surveyors Association. He received his B.S. Civil Engineering degree from the University of Illinois at Urbana-Champaign.

Todd.Horton.PE.PLS@gmail.com
(217) 493-3371 mobile

thorton@parkland.edu
(217) 373-3785 office
Relative Positional Precision Explained in Everyday Language

Todd Horton, PE, PLS
for
Wisconsin Society of Land Surveyors
January 2018

Truth versus Address

- Physical monument
  - Truth
  - Relatively stable

- Point coordinate
  - Merely an address
  - Contains error
  - Subject to change

American Land Title Association (ALTA) and National Society of Professional Surveyors (NSPS)

MINIMUM STANDARD DETAIL REQUIREMENTS FOR ALTA/NSPS LAND TITLE SURVEYS

(Effective February 23, 2016)
Allowable RPP

0.07 feet + 50 ppm

Application of ALTA Standards

E. Measurement Standards - The following measurement standards address Relative Positional Precision for the monuments or witnesses marking the corners of the surveyed property.

Two types of RPP

- For each possible line, you must calculate 2 relative positional precisions
  - Actual RPP
    - Generated by least squares analysis
  - Allowable RPP
    - Based solely on line length
Actual RPP Defined

“Relative Positional Precision” means the
- length of the semi-major axis, expressed in feet or meters,
- of the error ellipse representing the uncertainty due to random errors
- in measurements in the location of the monument, or witness, marking any corner of the surveyed property
- relative to the monument, or witness, marking any other corner of the surveyed property
- at the 95 percent confidence level.

WI ADMIN CODE, Chapter A−E 7
MINIMUM STANDARDS FOR PROPERTY SURVEYS

A−E 7.06 Measurements.
(3) The maximum allowable deviation in relative positional accuracy for a survey is plus or minus 0.07 foot plus 50 parts per million, based on the direct distance between the two corners being tested.
**Relative Positional Precision Explained in Everyday Language**

---

**Actual RPP**

RPP is not expressed at a point.

The size of the RPP error ellipse is a function of the error ellipses at the two end points of a line.

---

**RPP Evaluation**

“...the location of any corner relative to any other corner of the surveyed property...”

ALTA requires RPP evaluation for boundary lines and all possible lines between corners.

---

**Least Squares gives Actual RPP**

Relative Positional Precision is estimated by the results of a correctly weighted least squares adjustment of the survey.
Maximum Allowable RPP

The maximum allowable RPP for an ALTA/NSPS Land Title Survey is
• 2 cm (0.07 feet) plus 50 parts per million
• (based on the direct distance between the two corners being tested).

Allowable RPP

Allowable RPP is based on line length.

Boundary corner

Allowable RPP = \(0.12 \text{ ft}\)

\(0.07 \text{ ft} + \left(\frac{50}{1,000,000}\right) \times 1000.00 \text{ ft} = 0.12 \text{ ft}\)

(2 cm (0.07 feet) plus 50 parts per million)

Satisfying ALTA Requirements

• For each possible line, you must calculate 2 relative positional precisions
  – Actual RPP
    • Generated by least squares analysis
  – Allowable RPP
    • Based solely on line length
Actual versus Allowable

Actual RPP < Allowable RPP

It is recognized that in certain circumstances, the maximum allowable Relative Positional Precision may be exceeded.

If the maximum allowable Relative Positional Precision is exceeded, the surveyor shall note the reason as explained in Section 6.B.ix.

WI ADMIN CODE, Chapter A−E 7
MINIMUM STANDARDS FOR PROPERTY SURVEYS

A−E 7.06 Measurements. (3)

In certain circumstances, the size or configuration of the surveyed property, or the relief, vegetation or improvements on the surveyed property will result in survey measurements for which the maximum allowable relative positional precision may be exceeded at the discretion of the licensee performing the survey. The licensee shall provide justification for exceeding the maximum allowable relative positional accuracy.

Accuracy versus Precision

• Accuracy:
  - agreement of observed values with the "true value".
  - A measure of results.

• Precision:
  - agreement among readings of the same value (measurement).
  - A measure of methods.
Error & Accuracy

- The terms *error* and *accuracy* can be used to describe a single condition from opposite perspectives.

Error Defined

- the difference between an observed or calculated value and a true value;
  - specifically: variation in measurements, calculations, or observations of a quantity due to mistakes or to uncontrollable factors

- the amount of deviation from a standard or specification

**Mistake ≠ Error**

- Mistake - a gross error or blunder resulting usually from misunderstanding, carelessness or poor judgment.
- Error - The difference between a measured or calculated value and the true value.

**Systematic Errors**

- Errors that conforms to mathematical and physical laws.
  - Can be measured / quantified
  - Have a positive or negative value
  - Can be determined and/or corrected by procedure
- Examples
  - Incorrect prism offset
  - Erroneous EDM settings
  - Incorrect rod height

**Random Error**

- Errors that remain after mistakes and systematic errors have been eliminated.
  - Caused by factors beyond observers' control
  - Cannot be measured / quantified
  - Tend to be small and compensating
  - Governed by laws of probability
Compensating Errors

<table>
<thead>
<tr>
<th>Unrounded values</th>
<th>Values rounded up</th>
<th>Rounded to nearest even</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.015 ft</td>
<td>1.02 ft</td>
<td>1.02 ft</td>
</tr>
<tr>
<td>1.025 ft</td>
<td>1.03 ft</td>
<td>1.02 ft</td>
</tr>
<tr>
<td>1.035 ft</td>
<td>1.04 ft</td>
<td>1.04 ft</td>
</tr>
<tr>
<td>1.045 ft</td>
<td>1.05 ft</td>
<td>1.04 ft</td>
</tr>
<tr>
<td>1.055 ft</td>
<td>1.06 ft</td>
<td>1.06 ft</td>
</tr>
<tr>
<td>1.065 ft</td>
<td>1.07 ft</td>
<td>1.06 ft</td>
</tr>
<tr>
<td>Σ = 6.240 ft</td>
<td>Σ = 6.27 ft</td>
<td>Σ = 6.24 ft</td>
</tr>
</tbody>
</table>

Rounding up produced a cumulative 0.03 ft error.

Nature of Random Errors

- Positive and negative errors will occur with the same frequency.
- Minor errors will occur more often than large ones.
- Very large errors will rarely occur.

Managing Errors

- Mistake
  - Must be removed
- Systematic
  - Must be corrected
- Random
  - Must be minimized
Introduction to Adjustments

Adjustment:
“A process designed to remove inconsistencies in measured or computed quantities by applying derived corrections to compensate for random, or accidental errors, such errors not being subject to systematic corrections”.

Compass Rule (Bowditch Rule)
- Assumes angles and distances are measured with equal accuracy.
- Applies distance corrections in proportion to traverse course lengths.

Transit Rule
- Assumes angles are measured more accurately than distances.
- Applies latitude & departure corrections in proportion to traverse course latitudes & departures.
Crandall's Rule

- Quasi-statistical approach.
- Angles are held and errors are statistically distributed into the distances.
- Applies a least squares adjustment to each course length.

Least Squares Adjustment

- Allows full random error modeling.
- Can mix different accuracy and precision measurements.
- Provides measurement uncertainties.

Least Squares gives Actual RPP

Relative Positional Precision is estimated by the results of a correctly weighted least squares adjustment of the survey.
Least Squares Is ...

- A rigorous statistical adjustment of survey data based on the laws of probability and statistics.
- Provides simultaneous adjustment of all measurements.
- Measurements can be individually weighted to account for different error sources and values.

Least Squares Is Not ...

- A way to correct a weak strength of figure
- A cure for sloppy surveying - Garbage in / Garbage out
- The only adjustment available to the land surveyor

Use Least Squares for ...

Adjustment of:
- Conventional Traverse
- Control Networks
- GPS Networks
- Level Networks
- Resections

Collected By:
- Theodolite & Tape
- Total Stations
- GPS Receivers
- Levels
- EDMs
What Happens in Least Squares?

- Iterative Process
- Each iteration applies adjustments to observations, working for best solution
- Adjustments become smaller with each successive iteration

Requirements for Least Squares

- Redundant measurements
- Some controls (constraints)
- Precision estimates for measurements

Basic Error Computations
Measurement Scenario

• Measure a line that is very close to 100 feet long and determine the precision of your measurement.

Measures of Central Tendency

• The value within a data set that tends to exist at the center.
  – Arithmetic Mean
  – Median
  – Mode

Average Result

• Most commonly used is Arithmetic Mean
• Considered the “most probable value”

\[
\text{mean} = \frac{\sum \text{meas.}}{n} \\
\text{n = number of observations}
\]

• Mean = 1000 / 10
• Mean = 100.00’
### Measured Data Set

<table>
<thead>
<tr>
<th>Measured Data Set</th>
<th>Mean = 100.00 ft</th>
<th>Median = 99.98 ft</th>
<th>Mode = 100.00 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.96</td>
<td>100.02</td>
<td>99.98</td>
<td></td>
</tr>
<tr>
<td>100.04</td>
<td>100.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100.00</td>
<td>100.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100.02</td>
<td>100.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>99.98</td>
<td>100.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100.00</td>
<td>99.98</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100.00</td>
<td>99.96</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **Mean** = 100.00 ft
- **Median** = 100.00 ft
- **Mode** = 100.00 ft

### Standard Deviation

- Standard Deviation is a comparison of the individual readings (measurements) to the mean of the readings.
- Therefore, Standard Deviation is a measure of **PRECISION**

### Residuals

- The difference between an individual reading in a set of repeated measurements and the mean.
  - Residual $(v) = \text{reading} - \text{mean}$
  - Sum of the residuals squared $(\sum v^2)$ is used in future calculations.
Relative Positional Precision Explained in Everyday Language

<table>
<thead>
<tr>
<th>Readings</th>
<th>Residual</th>
<th>(Residual)^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.96</td>
<td>-0.04</td>
<td>0.0016</td>
</tr>
<tr>
<td>100.02</td>
<td>+0.02</td>
<td>0.0004</td>
</tr>
<tr>
<td>100.04</td>
<td>+0.04</td>
<td>0.0016</td>
</tr>
<tr>
<td>100.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>99.98</td>
<td>-0.02</td>
<td>0.0004</td>
</tr>
<tr>
<td>100.02</td>
<td>+0.02</td>
<td>0.0004</td>
</tr>
<tr>
<td>100.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>99.98</td>
<td>-0.02</td>
<td>0.0004</td>
</tr>
<tr>
<td>100.00</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \Sigma v = 0 \]
\[ \Sigma v^2 = 0.0048 \]

\[
\text{Standard Deviation Formula} \\
\sigma = \pm \sqrt{\frac{\Sigma v^2}{n-1}} = \pm 0.023' \\
\sigma = \pm \sqrt{\frac{0.0048}{9}} = \pm 0.023' 
\]

Measurement Components

- Each measurement consists of two components.

100.00 ft ± 0.023 ft

- The uncertainty statement is not a guess but is an estimate of precision calculated as standard deviation.
- 100 measurements of 1 distance
- Mean = 2000.00 feet
- Standard deviation = ±0.10 feet

- 100 measurements of 1 distance
- Mean = 2000.00 feet
- Standard deviation = ±0.04 feet

σ = ±0.023'
Normal Distribution

- Positive and negative errors will occur with the same frequency.
- Area under curve is equal on either side of the mean.

Minor Errors

- Minor errors will occur more often than large ones.
- The area within one standard deviation ($\sigma$) of the mean is 68.3% of the total area.

Large Errors

- Very large errors will rarely occur.
- The total area within 2$\sigma$ of the mean is 95% of the sample population.
Standard Deviation Probability

- The Standard Deviation is the ± range within which 68.3% of the residuals will fall or ...
- Each residual has a 68.3% probability of falling within the Standard Deviation range or ...
- If another measurement is made, the resulting residual has a 68.3% chance of falling within the Standard Deviation range.

Confidence Levels

- 68%
- 95%
- 99%

Expressions of Error

- E is a general term for error.
- These are specific terms for error.
  - CEP
  - Standard Deviation
  - 90%, 95%, 99% errors
95% Error

- 95% level of certainty is normal for surveying applications.

\[ E_{95} = \pm (1.96 \sigma) \]

95% Error

- Given 100.00 ft ± 0.023 ft (68% confidence)
  
  \[ E_{95} = \pm (2\sigma) = \pm (2 \times 0.023) = \pm 0.046' \]

- Distance = 100.00 ft ± 0.046 ft (95% confidence)

Meaning of \( E_{95} \)

"If a measurement falls outside of two standard deviations, it isn’t a random error, it’s a mistake!"

Francis H. Moffitt
How Errors Propagate

- Errors of a Series
- Errors of a Sum
- Error of Redundant Measurement

Error of a Series

\[ E_{\text{series}} = \pm E \sqrt{n} \]

- Describes the error of multiple measurements with identical standard deviations, such as measuring a 1000’ line with using a 100’ steel tape.

\[ E_{\text{series}} \]

Application

- A control level circuit of 64 rod readings was run between two benchmarks.
- All readings were made to the nearest 0.01 ft; the error in each reading was ±0.005 ft.
- For reading errors only, what total error would be expected in the elevation of the ending benchmark?
**E\text{\textsubscript{series}} Solution**

- E = ±0.005 ft
- n = 64 readings
  \[ E\text{\textsubscript{series}} = ±0.005\sqrt{64} \]
- E\text{\textsubscript{series}} = ±0.040 ft
  - Elevation error at ending benchmark

**Constant & Scalar Errors**

RTK horizontal error: ±10mm + 1ppm

Scalar error increases with distance.

**Error of a Sum**

\[ E\text{\textsubscript{sum}} = ±\sqrt{(E^2_1 + E^2_2 + E^2_3 + ... + E^2_n)} \]

- \( E\text{\textsubscript{sum}} \) is the square root of the sum the errors of each of the individual measurements squared.
- It is used when there are multiple unique error sources with differing standard deviations.
**E\text{sum} Application**

- You have measured the distance of 3609.14 feet using a total station and fixed targets.
- Your EDM has an accuracy of ±(3 mm + 3 ppm).
- Your instrument centering error is ±2 mm.
- Your target centering error is ±4 mm.

**Scalar Error**

- EDM accuracy = ±(3 mm + 3 ppm).
  - 3 mm = constant error
  - 3 ppm = scalar error (distance dependent)

- Convert distance to metric
  - 3609.14 ft x (12/39.37) m/ft = 1100.068 m

- 3 ppm = 3/1000000 x distance
  - 3/1000000 x 1100.068 m = 0.0033 m
  - 0.0033 m = 3.3 mm

**E\text{sum} Solution**

- Compute the standard deviation for the measured line.

\[ E_{\text{sum}} = \pm \sqrt{(2 \text{ mm}^2 + 4 \text{ mm}^2 + 3 \text{ mm}^2 + 3.3 \text{ mm}^2)} \]

- \[ E_{\text{sum}} = \pm 6.3 \text{ mm} \]
**E_{sum} Solution**

- Distance = 1100.068 m ± 6.3 mm
- Distance = 3609.14 ft ± 0.021 ft

**Relative Positional Precision**

RPP is not expressed at a point. The size of the RPP error ellipse is a function of the error ellipses at the two end points of the line.

**Actual RPP (approximation)**

\[ E_{sum} = \pm \sqrt{(E_1^2 + E_2^2 + E_3^2 + \ldots + E_n^2)} \]

\[ RPP = \pm \sqrt{(0.037^2 + 0.061^2)} \]

\[ RPP = \pm 0.071 \text{ ft} \]
Allowable RPP

0.07 feet + 50 ppm

ALTA allowable RPP

• What is the allowable ALTA RPP for a line of 1,000 feet?
  – Constant component = 0.07 ft
  – Scalar component = 50 ppm
    • 1000 ft X (50/1,000,000) = 0.05 feet
  
  • RPP = 0.07 + 0.05 = 0.12 ft

  Note: the constant and scalar components are not random errors, so the error of a sum does not apply.

What if RPP goes bad?

• If actual RPP exceeds the allowable RPP, decrease the size of the error ellipses at the endpoints of the line in question.

• How?
  – make redundant measurements
  – use higher precision equipment
  – develop a different survey design
RPP Keys for Success

1. Choose equipment and measurement methods based on accuracy needs of the project.

Error management

- Some amount of error is acceptable.
  - Acceptable error is determined by the intended use of the measurement.
  - ALTA/NSPS standards define allowable limits.
- Good surveying procedures are designed to:
  - Remove mistakes
  - Correct for systematic errors
  - Minimize random errors.

Standard errors

- A standard error is an estimated error expected from a particular field procedure and equipment application.
  - Instrument angular error
  - EDM errors, constant & scalar
  - Centering errors
**EDM Error Sources**

- **Systematic**
  - Incorrect reflector constant
  - Instrument calibration
    - Constant & scalar error
  - Temperature
  - Atmospheric pressure
- **Random**
  - Unequal refraction
  - Tripod settlement
  - Instrument miscentering
  - Target miscentering

**Angular Measurement Error Sources**

- **Systematic**
  - Maladjustment of plate bubble
  - Horizontal and vertical axes not perpendicular
  - Horizontal and sight axes not perpendicular
  - Vertical circle indexing error
  - Eccentricity of centers
  - Circle graduation errors
- **Random**
  - Pointing
  - Unequal refraction
  - Tripod settlement
  - Parallax
  - Wind effects
  - Centering
Angular Errors

- Instrument angle least count ≠ accuracy
- Instrument angular accuracy specified by DIN 18723
  - Deutsches Institut für Normung
  - German Institute for Standards
  - http://www.din.de/

DIN 18723

<table>
<thead>
<tr>
<th>DIN 18723 Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>5&quot; TS</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Faces</td>
</tr>
<tr>
<td>Direct only</td>
</tr>
<tr>
<td>1 set D &amp; R</td>
</tr>
<tr>
<td>2 sets D &amp; R</td>
</tr>
</tbody>
</table>
### Instrument Specifications

- **Trimble 5600 Series total stations**

<table>
<thead>
<tr>
<th>Angle Measurement accuracy</th>
<th>5602</th>
<th>5603</th>
<th>5605</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy (Standard Deviation based on DIN 18723)</td>
<td>2”</td>
<td>3”</td>
<td>5”</td>
</tr>
<tr>
<td>Angle reading (least count)</td>
<td>1”</td>
<td>1”</td>
<td>1”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Distance Measurement accuracy</th>
<th>5602</th>
<th>5603</th>
<th>5605</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic mean value</td>
<td>±(2 mm + 2 ppm)</td>
<td>±(2 mm + 2 ppm)</td>
<td>±(3 mm + 3 ppm)</td>
</tr>
<tr>
<td>Standard measurement</td>
<td>±(3 mm + 2 ppm)</td>
<td>±(3 mm + 2 ppm)</td>
<td>±(5 mm + 3 ppm)</td>
</tr>
<tr>
<td>Fast standard</td>
<td>±(8 mm + 2 ppm)</td>
<td>±(8 mm + 2 ppm)</td>
<td>±(8 mm + 3 ppm)</td>
</tr>
<tr>
<td>Fast tracking</td>
<td>±(10 mm + 2 ppm)</td>
<td>±(10 mm + 2 ppm)</td>
<td>±(10 mm + 3 ppm)</td>
</tr>
</tbody>
</table>

### GNSS Error Sources

- Clock synchronization error
- Orbital error
  - Predicted versus as-flown trajectories
  - Dilution of precision
- Multipath error
- Atmospheric error
Good PDOP

GNSS Error: Multipath

Reflected signals yield poor accuracy.

GNSS Errors: Atmospheric Delay

12,500 mi
125 mi
31 mi

Ionosphere
Troposphere
RTK Positional Error

RTK positional error increases with distance from the base.

Trimble R8 Accuracy Specs

<table>
<thead>
<tr>
<th>Method</th>
<th>Constant Error</th>
<th>Scalar Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Code differential GRSS position</td>
<td>5 mm + 0.1 ppm RMS</td>
<td>5 mm + 0.4 ppm RMS</td>
</tr>
<tr>
<td>WAAS differential positioning</td>
<td>5 mm + 0.1 ppm RMS</td>
<td>5 mm + 0.4 ppm RMS</td>
</tr>
<tr>
<td>Static and Fast Static GNSS surveying</td>
<td>10 mm + 1 ppm RMS</td>
<td>20 mm + 1 ppm RMS</td>
</tr>
<tr>
<td>Kinematic surveying</td>
<td>10 mm + 1 ppm RMS</td>
<td>20 mm + 1 ppm RMS</td>
</tr>
</tbody>
</table>

- Initialization time: typically <10 seconds
- Initialization reliability: typically >99.9%


RTK Vector Errors

- Vector is the line from base to rover.
  - 10 mm + 1 ppm horizontal error for RTK vectors is typical.
  - 10 mm = 0.033 ft = constant error
  - 1 ppm = scalar error (distance dependent)
    - 1 part error to 1,000,000 parts measurement
    - 1 mm error / 1 km RTK vector
    - 0.005 ft error / 1 mile RTK vector
RTK Vector Errors

\[ E_{\text{dist}} = \sqrt{E_{\text{const}}^2 + E_{\text{scalar}}^2} \]

<table>
<thead>
<tr>
<th>RTK vector (miles)</th>
<th>E_{\text{const}} (feet)</th>
<th>E_{\text{scalar}} (feet)</th>
<th>E_{\text{dist}} (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.033</td>
<td>0.005</td>
<td>0.033</td>
</tr>
<tr>
<td>2</td>
<td>0.033</td>
<td>0.011</td>
<td>0.034</td>
</tr>
<tr>
<td>4</td>
<td>0.033</td>
<td>0.021</td>
<td>0.039</td>
</tr>
<tr>
<td>8</td>
<td>0.033</td>
<td>0.042</td>
<td>0.053</td>
</tr>
<tr>
<td>16</td>
<td>0.033</td>
<td>0.084</td>
<td>0.091</td>
</tr>
<tr>
<td>32</td>
<td>0.033</td>
<td>0.169</td>
<td>0.172</td>
</tr>
</tbody>
</table>

RTK relative accuracy

RTK relative accuracy is independent of base station absolute accuracy.

TS vs GNSS comparison

Vector

\[ d\text{RMS} \]
3 second TS versus GNSS

![Graph showing dRMS: Total Station vs GNSS]

- Trimble T5603: 3 seconds, 3mm+2ppm
- Trimble R8: 10mm+1ppm RTK horizontal

**dRMS: Total Station vs GNSS**

- Single angles
- 1 set D&R
- 2 sets D&R
- 3 sets D&R
- RTK GNSS

**Vector Length (feet)**

**dRMS (feet)**

**Match the tool to the task**

- **GPS methods**
  - give greater accuracy over long distances.

- **Total station methods**
  - give greater accuracy over shorter distances.

**RPP Keys for Success**

- **Use well adjusted instruments and procedures to eliminate systematic errors in measurements.**
Relative Positional Precision
Explained in Everyday Language

Line of Sight Error

The line-of-sight error, or horizontal collimation error is the deviation from the perpendicular between the tilting axis and the line of sight. The effect of the line-of-sight error to the horizontal direction increases with the vertical angle.

a: Tilting axis
b: Line perpendicular to tilting axis
c: Horizontal collimation, or line-of-sight, error
d: Line-of-sight

Vertical Index Error

The vertical circle should read exactly 90° when the line of sight is horizontal. Any deviation from this figure is termed vertical index error. This is a constant error that affects all vertical angle readings.

a: Mechanical vertical axis of the instrument
b: Axis perpendicular to vertical axis. True 90°
c: Vertical angle is reading 90°
d: Vertical index error

Tilting Axis Error

The tilting axis error is caused by the deviation between the mechanical tilting axis and the line perpendicular to the vertical axis. This error affects horizontal angles. To determine this error, it is necessary to point to a target located significantly below or above the horizontal plane.
Tribrach Optical Plummet

RPP Keys for Success

3 Make internal checks to detect blunders in measurements.

Blunder Check

Redundant measurements are those measurements in excess of the minimum number needed to determine the unknown coordinates.
Total Station Blunder Prevention

- Measure network angles as sets
  - Direct and reverse
  - Multiple sets
- Measure all traverse leg distances twice
  - Foresight and backsight distances
  - Use average distance

GNSS Blunder Prevention

- Repeat the RTK observation using a new base location.
- Repeat the RTK observation using a new base instrument height.
- Repeat the observation with base and rover receivers swapped.

Independent Methods

- Check GNSS distances with total station.
- Check total station distances with GNSS.
- Check GNSS elevation differences with:
  - Total station
  - Leveling
- Check total stations elevation differences with:
  - GNSS
  - leveling
RPP Keys for Success

4

Make redundant measurements to have a large degree of freedom.

Degree of freedom

- The degree of freedom is an indication of how many redundant measurements are in the survey.
- Degree of freedom is defined as the number of measurements in excess of the number of measurements necessary to solve the network.

Statistics

<table>
<thead>
<tr>
<th>Statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Observations: 271</td>
<td></td>
</tr>
<tr>
<td>Total Hyps: 184</td>
<td></td>
</tr>
<tr>
<td>Degrees of Freedom: 87</td>
<td></td>
</tr>
<tr>
<td>Observation Count</td>
<td></td>
</tr>
<tr>
<td>Angles: 132</td>
<td></td>
</tr>
<tr>
<td>Distances: 99</td>
<td></td>
</tr>
<tr>
<td>Total: 271</td>
<td></td>
</tr>
</tbody>
</table>

Redundant measurements

- Running additional cut-off traverses, or additional traverses to existing control points, creates redundancy.
Redundant measurements

Measuring points in the network that have already been located creates redundancy.

Error in Redundant Measurements

If a measurement is repeated multiple times, the accuracy increases, even if the measurements have the same value.

\[ E_{\text{red.meas.}} = \pm \frac{E}{\sqrt{n}} \]

Error in Redundant Measurements

- With Errors of a Sum (or Series), each additional variable *increases* the total error of the network.

- With Errors of Redundant Measurement, each redundant measurement *decreases* the error of the network.
Redundant Measurement Example

- Example: Angle measured with 5" total station

\[ E_{\text{sum}} = \pm \sqrt{(E_1^2 + E_2^2 + E_3^2 + \ldots + E_n^2)} \]
- 5" error in direct & reverse pointing at BS
- 5" error in direct & reverse pointing at FS

\[ E_{\text{sum}} = \pm \sqrt{(5^2 + 5^2)} = \pm 7" \]
- Standard error in 1 set, direct & reverse

Redundant Measurement Example

- Example: Angle measured with 5" total station
- Standard error in 1 set, direct & reverse

- 1 set \[ E_{\text{sum}} = \pm \sqrt{(5^2 + 5^2)} = \pm 7" \]
- 2 sets \[ E_{\text{red meas}} = \pm \frac{E}{\sqrt{n}} = \pm \frac{7"}{\sqrt{2}} = \pm 5" \]
- 3 sets \[ E_{\text{red meas}} = \pm \frac{E}{\sqrt{n}} = \pm \frac{7"}{\sqrt{3}} = \pm 4" \]
- 4 sets \[ E_{\text{red meas}} = \pm \frac{E}{\sqrt{n}} = \pm \frac{7"}{\sqrt{4}} = \pm 3.5" \]

Complexity & Redundancy

As a network becomes more complex, maintain accuracy by increasing the number of redundant measurements.
Adding Redundancy: GNSS

• Repeat the observation with different satellite geometry.
  – Re-measure control points before, during, and after survey session.

Adding Redundancy: Total Station

• Measure network angles as sets
  – Direct and reverse
  – Multiple sets

• Measure all traverse leg distances twice
  – Foresight and backsight distances
  – Use average distance

Satellite geometry changes.
RPP Keys for Success

Assess the precision of measurements accurately.

Error Ellipses

- Used to described the accuracy of a measured survey point.
- Error Ellipse is defined by the dimensions of the semi-major and semi-minor axis and the orientation of the semi-major axis.
- Assuming standard errors, the measurements have a 39.4% chance of falling within the Error Ellipse.

<table>
<thead>
<tr>
<th>Error Ellipses</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Northing standard deviation</td>
<td>Measured N 583,511.32</td>
</tr>
<tr>
<td>Easting standard deviation</td>
<td>E 2,068,582.47</td>
</tr>
<tr>
<td>Semi-major axis</td>
<td>39% confidence</td>
</tr>
<tr>
<td>Semi-minor axis</td>
<td></td>
</tr>
</tbody>
</table>
Relative Positional Precision
Explained in Everyday Language

Error Ellipse versus dRMS

\[ dRMS = \sqrt{E_1^2 + E_2^2} \]

Radius = dRMS
\approx 65\% confidence

Distance Root Mean Square

\approx 65\% confidence

- HRMS
  - Trimble
  - Topcon
- 2DCQ
  - Leica

Reported Precision

1dRMS
Displayed at data collector

2dRMS
Specified in contracts
Reported in NGS datasheets
Error circles: 1dRMS & 2dRMS

- Contrary to one-dimensional statistics, there is no fixed probability level for this error measure.
- The confidence level depends on the ratio of standard deviations.
- Owing to the low probability content of the dRMS error circle, 95% is generally required for position-finding errors.

1dRMS & 2dRMS

<table>
<thead>
<tr>
<th>(\sigma_y/\sigma_x)</th>
<th>1*dRMS</th>
<th>Confidence @ 1*dRMS</th>
<th>2*dRMS</th>
<th>Confidence @ 2*dRMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.0</td>
<td>68.27%</td>
<td>2.0</td>
<td>95.45%</td>
</tr>
<tr>
<td>0.25</td>
<td>1.0308</td>
<td>68.15%</td>
<td>2.0616</td>
<td>95.91%</td>
</tr>
<tr>
<td>0.5</td>
<td>1.1180</td>
<td>66.29%</td>
<td>2.2361</td>
<td>96.97%</td>
</tr>
<tr>
<td>0.75</td>
<td>1.25</td>
<td>63.92%</td>
<td>2.5</td>
<td>97.87%</td>
</tr>
<tr>
<td>1.0</td>
<td>1.4142</td>
<td>63.20%</td>
<td>2.8284</td>
<td>98.16%</td>
</tr>
</tbody>
</table>

Confidence at 1dRMS

- Increasing confidence
  - \(\sigma_y/\sigma_x = 1.0\)
    - 63.20% 68.27%
  - \(\sigma_y/\sigma_x = 0.75\)
    - 63.92% 68.15%
  - \(\sigma_y/\sigma_x = 0.50\)
    - 66.29% 66.29%
  - \(\sigma_y/\sigma_x = 0.25\)
    - 68.27% 68.27%
Confidence at 2dRMS

Confidence Levels

Relative & Absolute

- An absolute error ellipse indicates the error of a single point relative to the datum.

- A relative error ellipse is a statistical measure of the expected error between two points.
  - Survey accuracy standards commonly state the maximum allowable error between any two points in a survey.
  - Relative error ellipses can give this information.
Relative Positional Precision
Explained in Everyday Language

Absolute & relative accuracy

A: known
B: unknown

Absolute accuracy = ±0.07'
Relative accuracy = ±0.05'

Absolute accuracy = ±5'
Relative accuracy = ±0.05'

Poor absolute accuracy
Good relative accuracy

Good absolute accuracy
Good relative accuracy

RPP Keys for Success

6
Assess the quality of control which will be used.
Old: Order & Class Codes

LC1765
LC1765 CBN — This is a Cooperative Base Network Control Station.
LC1766 DESIGNATION - ALEXANDER 2
LC1765 PID - LC1766
LC1766 STATE/COUNTY - IL/PIATT
LC1766 USGS QUAD - SEYMOUR (1970)
LC1766
LC1766 *CURRENT SURVEY CONTROL
LC1766
LC1766 NAD 83(1997)- 40 06 20.45303(N) 088 29 42.90776(W) ADJUSTED
LC1766 NAVD 88 - 231.3 (meters) 759 (feet) GPS OBS
LC1766
LC1766 X - 128,287.167 (meters) COMP
LC1766 Y - -4,883,624.381 (meters) COMP
LC1766 Z - 4,087,096.112 (meters) COMP
LC1766 LAPLACE CORR- -0.58 (seconds) DEFLEC99
LC1766 ELLIP HEIGHT- 199.36 (meters) GPS OBS
LC1766 GEOID HEIGHT- -31.94 (meters) GEOID99

Current: Numeric Accuracy

LC1765
LC1765 CBN — This is a Cooperative Base Network Control Station.
LC1766 DESIGNATION - ALEXANDER 2
LC1765 PID - LC1766
LC1766 STATE/COUNTY - IL/PIATT
LC1766 COUNTRY - US
LC1766 USGS QUAD - SEYMOUR (1970)
LC1766
LC1766 *CURRENT SURVEY CONTROL
LC1766
LC1766 NAD 83(2011) POSITION- 40 06 20.45302(N) 088 29 42.90701(W) ADJUSTED
LC1766 NAD 83(2011) ELLIP HT- 199.346 (meters) (06/27/12) ADJUSTED
LC1766 NAD 83(2011) EPOCH - 2010.00
LC1766
LC1766 NAVD 88 ORTHO HEIGHT - 231.3 (meters) 759 (feet) GPS OBS

FGDC Geospatial Positioning Accuracy Standards (95% confidence, cm)

<table>
<thead>
<tr>
<th>Type</th>
<th>Horiz Ellip Dist (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NETWORK</td>
<td>0.77</td>
</tr>
<tr>
<td>MEDIAN LOCAL ACCURACY AND DIST (039 points)</td>
<td>2.21</td>
</tr>
<tr>
<td>2dRMS = 0.77 cm</td>
<td></td>
</tr>
<tr>
<td>95% confidence</td>
<td></td>
</tr>
</tbody>
</table>

Control

- Occupy at least one control point.
- Control points need not be adjacent to each other.
Control

- It is permissible to have one control point on one side of the project and a reference direction on the other side of the project.

RPP Keys for Success

7

Avoid weak network geometry.

Poor strength of figure

Figure lacks rigidity.
Poor strength of figure

Figure still lacks rigidity.

Added line

Poor strength of figure

Figure still lacks rigidity.

Added line

Triangulation Network Example

Triangles provide rigidity.
Good strength of figure

Triangles provide rigidity.

Good PDOP

Volume of enclosed figure is high.

Poor PDOP, Poor HDOP

Volume of enclosed figure is low.

Good VDOP
Relative Positional Precision Explained in Everyday Language

Poor PDOP, Poor VDOP

Volume of enclosed figure is low.

Good HDOP

RPP Keys for Success

8
Organize all field measurements for software input.

Flexibility

- Least squares is very flexible in terms of how the survey data needs to be collected.
- Generally speaking, any combination of angles and distances, combined with a minimal amount of control points and azimuths, are needed.
- This data can be collected in any order.
Preliminary Control

• There may be situations where no control point is ever occupied in the network, but only backsighted.

• In these situations, a preliminary value for one of the occupied points needs to be computed and entered as a floating point control point.

Control Data Management

• It is always best to explicitly define the control for the project.

• A good method is to put all the control for a project into a separate raw file.

• A big source of problems with new users is a misunderstanding in defining their control for a project.

Point Number Management

• The majority of all problems in processing raw data are related to point number problems.
  – Using the same point number twice for different points
  – Not using the same point number when shooting the same point
Inspect data for obvious blunders.

• Misnumbering backsights or foresights
• Misnumbering control points

RPP Keys for Success

Establish standard errors for all observation conditions.

Standard errors

• A standard error is an estimated error expected from a particular field procedure and equipment application.
  – Instrument angular error
  – EDM errors, constant & scalar
  – Centering errors

<table>
<thead>
<tr>
<th>Static and Fast Static GNSS surveying¹</th>
<th>Horizontal: 3 mm + 0.1 ppm RMS</th>
<th>Vertical: 3.5 mm + 0.4 ppm RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinematic surveying¹</td>
<td>Horizontal: 10 mm + 1 ppm RMS</td>
<td>Vertical: 20 mm + 1 ppm RMS</td>
</tr>
</tbody>
</table>
**Definition: A Priori**

- Latin for “from what comes before”.
- In the sense in which it is used in the subject of probability, it means
  - Derived by logic, without observed facts
  - Involving deductive reasoning from a general principle to a necessary effect; not supported by fact
  - Based on hypothesis or theory rather than experiment

**Standard errors**

- Standard errors are an estimate of the different errors you would expect to obtain based on the type equipment and field procedures you used to collect the raw data.
  - For example, if you are using a 5 second theodolite, you could expect the angles to be measured within +/- 5 seconds (Reading error).

**Overriding standard errors**

- There are times when the default standard errors for points values may need to be overridden.
  - For example, the control may be from GPS and the user has differing standard errors for his various GPS points.
  - Or maybe some of the control points were collected with RTK methods, and other GPS points collected with more accurate static GPS methods.
Standard Error Input

Standard Errors, Traverse 1

- Distance
  - Constant: 0.010 ft
  - Scalar: 5 PPM

- Horizontal angle
  - Pointing: 3.0"
  - Reading: 3.0"

- Vertical angle
  - Pointing: 3.0"
  - Reading: 3.0"

Standard Errors, Traverse 1

- Total Station
  - Centering: 0.005 ft
  - Height: 0.010 ft

- Target
  - Centering: 0.005 ft
  - Height: 0.010 ft

- Azimuth: 5"

- Coordinate Control
  - N: 0.010 ft
  - E: 0.010 ft
  - Z: 0.020 ft
RPP Keys for Success

10 Adjust and analyze results.

Weight of Measurement

- The concept of weighting measurements to account for different error sources is fundamental to a least squares adjustment.

- Weighting can be based on error sources, if the error of each measurement is different, or the quantity of readings that make up a reading, if the error sources are equal.

Weight of Measurement

- Weight - "The relative reliability (or worth) of a quantity as compared with other values of the same quantity."

Definitions of Surveying and Associated Terms, 1989 Reprint
Weighted Adjustment

Perform a weighted adjustment based on this data:

- \( A = 43° 24' 36" \pm 5" \) with \( W = \frac{1}{5} = 0.2000 \)
- \( B = 47° 12' 34" \pm 15" \) with \( W = \frac{1}{15} = 0.0666 \)
- \( C = 89° 22' 20" \pm 30" \) with \( W = \frac{1}{30} = 0.0333 \)

\( W \propto \left( \frac{1}{\sigma} \right) \) (Error Sources)

<table>
<thead>
<tr>
<th>Angle</th>
<th>Measured value</th>
<th>Wt</th>
<th>Correction</th>
<th>Rounded correction</th>
<th>Adjusted value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>43° 24' 36&quot;</td>
<td>0.2000</td>
<td>( 0.3000 \times 0.2000 \times ) x</td>
<td>+3&quot;</td>
<td>43° 24' 39&quot;</td>
</tr>
<tr>
<td>B</td>
<td>47° 12' 34&quot;</td>
<td>0.0667</td>
<td>( 0.3000 \times 0.0667 \times ) x</td>
<td>+9&quot;</td>
<td>47° 12' 43&quot;</td>
</tr>
<tr>
<td>C</td>
<td>89° 22' 20&quot;</td>
<td>0.0333</td>
<td>( 0.3000 \times 0.0333 \times ) x</td>
<td>+18&quot;</td>
<td>89° 22' 38&quot;</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>179° 59' 30&quot;</td>
<td>0.3000</td>
<td>15.00x</td>
<td>30&quot;</td>
<td>180° 00' 00&quot;</td>
</tr>
</tbody>
</table>

\( 15.00x = 30" \)
\( x = 2" \)

What Happens in Least Squares?

- Iterative Process
- Each iteration applies adjustments to observations, working for best solution
- Adjustments become smaller with each successive iteration
Demonstration Project 1

Simple 2D Network
Traverse and Side Shots

Traverse 1

- 2 fixed control points
- 8 occupations
- Each angle measured once, direct only
- Data imported to software from data collector file

Standard Errors, Traverse 1

- Distance
  - Constant: 0.010 ft
  - Scalar: 5 PPM
- Horizontal angle
  - Pointing: 3.0"
  - Reading: 3.0"
- Vertical angle
  - Pointing: 3.0"
  - Reading: 3.0"
Standard Errors, Traverse 1

- **Total Station**
  - Centering: 0.005 ft  Height: 0.010 ft

- **Target**
  - Centering: 0.005 ft  Height: 0.010 ft

- **Azimuth:** 5°

- **Coordinate Control**
  - N: 0.010 ft  E: 0.010 ft  Z: 0.020 ft

Allowable RPP Settings

This dialog box allows the user to define the points to be included in the ALTA report processing.
## Adjusted Local Coordinates

### Traverse 1

<table>
<thead>
<tr>
<th>Sta.</th>
<th>N:</th>
<th>E:</th>
<th>StErr N:</th>
<th>StErr E:</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR2</td>
<td>5003.321</td>
<td>4132.516</td>
<td>0.023</td>
<td>0.011</td>
</tr>
<tr>
<td>TR1B</td>
<td>4781.616</td>
<td>4986.993</td>
<td>0.013</td>
<td>0.009</td>
</tr>
<tr>
<td>TR3</td>
<td>4198.753</td>
<td>4130.552</td>
<td>0.026</td>
<td>0.022</td>
</tr>
<tr>
<td>TR4</td>
<td>5706.868</td>
<td>4124.208</td>
<td>0.027</td>
<td>0.03</td>
</tr>
<tr>
<td>TR5</td>
<td>5702.894</td>
<td>4986.439</td>
<td>0.019</td>
<td>0.029</td>
</tr>
<tr>
<td>TR6</td>
<td>3705.783</td>
<td>5818.431</td>
<td>0.015</td>
<td>0.028</td>
</tr>
<tr>
<td>TR7</td>
<td>4191.594</td>
<td>5819.269</td>
<td>0.011</td>
<td>0.019</td>
</tr>
<tr>
<td>TR7B</td>
<td>4196.277</td>
<td>4986.248</td>
<td>0.037</td>
<td>0.025</td>
</tr>
</tbody>
</table>

## Adjusted Coordinate Error Ellipses

### Traverse 1

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Axis</td>
</tr>
<tr>
<td>TR2</td>
<td>0.087</td>
</tr>
<tr>
<td>TR1B</td>
<td>0.061</td>
</tr>
<tr>
<td>TR3</td>
<td>0.101</td>
</tr>
<tr>
<td>TR4</td>
<td>0.067</td>
</tr>
<tr>
<td>TR5</td>
<td>0.114</td>
</tr>
<tr>
<td>TR6</td>
<td>0.053</td>
</tr>
<tr>
<td>TR7</td>
<td>0.040</td>
</tr>
<tr>
<td>TR7B</td>
<td>0.139</td>
</tr>
</tbody>
</table>

## ALTA Tolerances, Traverse 1

*ALL CONNECTIONS: Tolerance of 0 073 + 50 PPM at the 95% CI*

All possible connections between the following points were checked.

TR2, TR1B, TR3, TR4, TR5, TR6, TR7, TR7B, TR3B, TR3B, TR7B

In connections tested, the 72 largest relative error ellipses will be shown.

<table>
<thead>
<tr>
<th>Sta.</th>
<th>Semi-Major</th>
<th>Semi-Minor</th>
<th>Allowable RPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR2</td>
<td>0.128</td>
<td>0.081</td>
<td>0.732</td>
</tr>
<tr>
<td>TR1B</td>
<td>0.123</td>
<td>0.064</td>
<td>0.964</td>
</tr>
<tr>
<td>TR3</td>
<td>0.123</td>
<td>0.064</td>
<td>0.964</td>
</tr>
<tr>
<td>TR4</td>
<td>0.123</td>
<td>0.064</td>
<td>0.964</td>
</tr>
<tr>
<td>TR5</td>
<td>0.123</td>
<td>0.064</td>
<td>0.964</td>
</tr>
<tr>
<td>TR6</td>
<td>0.123</td>
<td>0.064</td>
<td>0.964</td>
</tr>
<tr>
<td>TR7</td>
<td>0.123</td>
<td>0.064</td>
<td>0.964</td>
</tr>
<tr>
<td>TR7B</td>
<td>0.123</td>
<td>0.123</td>
<td>0.732</td>
</tr>
</tbody>
</table>

Allowable RPP is exceeded when ratio is greater than 1.
Adjustment Statistics, Traverse 1

Statistics
**********
Solution converged in 2 iterations
Total Observations: 20
Degrees of Freedom: 4
Observation Count | Sum Squares | Error of Estimate
Angles: 10 | 3.648 | 1.951
Distances: 10 | 0.444 | 0.471 (Horizontal)
Total: 20 | 4.092 | 1.011

Reference Variance: 1.023
Standard Error That Weight: [-*-] 0.011
Passed the Chi-Square test at the 95.00 significance level
0.484 (- 4.092 <- 11.143)

Indicator of redundancy

Adjustment Statistics, Traverse 2

Traverse 2

- 2 fixed control points
- 8 occupations
- Each angle measured in direct & reverse, 1 set
- Same standard errors as traverse 1

Adjusted Coordinate Error Ellipses

Traverse 2
95% confidence interval

<table>
<thead>
<tr>
<th>Sta.</th>
<th>Semi-Major Axis</th>
<th>Semi-Minor Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trav 1</td>
<td>Trav 2</td>
</tr>
<tr>
<td>TR2</td>
<td>0.087</td>
<td>0.044</td>
</tr>
<tr>
<td>TR1B</td>
<td>0.05</td>
<td>0.028</td>
</tr>
<tr>
<td>TR3</td>
<td>0.111</td>
<td>0.058</td>
</tr>
<tr>
<td>TR4</td>
<td>0.135</td>
<td>0.072</td>
</tr>
<tr>
<td>TR5</td>
<td>0.114</td>
<td>0.06</td>
</tr>
<tr>
<td>TR6</td>
<td>0.107</td>
<td>0.056</td>
</tr>
<tr>
<td>TR7</td>
<td>0.072</td>
<td>0.037</td>
</tr>
<tr>
<td>TR7B</td>
<td>0.139</td>
<td>0.072</td>
</tr>
</tbody>
</table>
ALTA Tolerances, Traverse 2

<table>
<thead>
<tr>
<th>Obs</th>
<th>Obs</th>
<th>Lat</th>
<th>Long</th>
<th>Error</th>
<th>Error</th>
<th>RATIO</th>
<th>Allowable</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR1</td>
<td>TR2</td>
<td>0.044</td>
<td>0.019</td>
<td>0.019</td>
<td>0.010</td>
<td>0.75</td>
<td>0.075</td>
</tr>
<tr>
<td>TR1</td>
<td>TR3</td>
<td>0.044</td>
<td>0.019</td>
<td>0.019</td>
<td>0.010</td>
<td>0.75</td>
<td>0.075</td>
</tr>
<tr>
<td>TR1</td>
<td>TR4</td>
<td>0.044</td>
<td>0.019</td>
<td>0.019</td>
<td>0.010</td>
<td>0.75</td>
<td>0.075</td>
</tr>
<tr>
<td>TR1</td>
<td>TR5</td>
<td>0.044</td>
<td>0.019</td>
<td>0.019</td>
<td>0.010</td>
<td>0.75</td>
<td>0.075</td>
</tr>
<tr>
<td>TR1</td>
<td>TR6</td>
<td>0.044</td>
<td>0.019</td>
<td>0.019</td>
<td>0.010</td>
<td>0.75</td>
<td>0.075</td>
</tr>
<tr>
<td>TR1</td>
<td>TR7</td>
<td>0.044</td>
<td>0.019</td>
<td>0.019</td>
<td>0.010</td>
<td>0.75</td>
<td>0.075</td>
</tr>
<tr>
<td>TR1</td>
<td>TR8</td>
<td>0.044</td>
<td>0.019</td>
<td>0.019</td>
<td>0.010</td>
<td>0.75</td>
<td>0.075</td>
</tr>
<tr>
<td>TR1</td>
<td>TR9</td>
<td>0.044</td>
<td>0.019</td>
<td>0.019</td>
<td>0.010</td>
<td>0.75</td>
<td>0.075</td>
</tr>
<tr>
<td>TR1</td>
<td>TR10</td>
<td>0.044</td>
<td>0.019</td>
<td>0.019</td>
<td>0.010</td>
<td>0.75</td>
<td>0.075</td>
</tr>
</tbody>
</table>

**Adjustment Statistics, Traverse 2**

**Statistics**
- Solution converged in 2 iterations

<table>
<thead>
<tr>
<th>Total Observations: 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Observations: 14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Degrees of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation Count</td>
</tr>
<tr>
<td>Sum of Squares of Stddes</td>
</tr>
<tr>
<td>Error Factor</td>
</tr>
<tr>
<td>Angles: 20</td>
</tr>
<tr>
<td>Distances: 10</td>
</tr>
<tr>
<td>Total: 30</td>
</tr>
<tr>
<td>0.640</td>
</tr>
<tr>
<td>0.376 (Horizontal)</td>
</tr>
<tr>
<td>13.497</td>
</tr>
<tr>
<td>0.982</td>
</tr>
</tbody>
</table>

Reference Variance: 0.964
Standard Error Unit Weight: (4/3)0.353
Passed the Chi-Square test at the 95.06 significance level
5.629 r = 13.497 r = 26.119

**Traverse 3**

- 2 fixed control points
- 8 occupations
- Each angle measured in direct & reverse, 2 sets
- Same standard errors as traverse 1
Adjusted Coordinate Error Ellipses

Traverse 3
95% confidence interval

<table>
<thead>
<tr>
<th>Sta.</th>
<th>Semi-Major Axis</th>
<th>Semi-Minor Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trav 1</td>
<td>Trav 3</td>
</tr>
<tr>
<td>TR2</td>
<td>0.087</td>
<td>0.027</td>
</tr>
<tr>
<td>TR1B</td>
<td>0.05</td>
<td>0.022</td>
</tr>
<tr>
<td>TR3</td>
<td>0.111</td>
<td>0.037</td>
</tr>
<tr>
<td>TR4</td>
<td>0.135</td>
<td>0.047</td>
</tr>
<tr>
<td>TR5</td>
<td>0.114</td>
<td>0.038</td>
</tr>
<tr>
<td>TR6</td>
<td>0.107</td>
<td>0.036</td>
</tr>
<tr>
<td>TR7</td>
<td>0.072</td>
<td>0.023</td>
</tr>
<tr>
<td>TR7B</td>
<td>0.139</td>
<td>0.045</td>
</tr>
</tbody>
</table>

ALTA Tolerances, Traverse 3

ALL CONNECTIONS: Tolerance of 0.870 + 50 PPM at the 95% CI.
All possible connections between the following points were checked:
TR1, TR2, TR3, TR4, TR5, TR6, TR7, TR7B, TR1B
N connections tested, the 99 percentile relative error ellipses will be shown

<table>
<thead>
<tr>
<th>Sta.</th>
<th>Sta.</th>
<th>Dist</th>
<th>$\Delta$</th>
<th>$\Delta$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR2</td>
<td>TR3</td>
<td>0.036</td>
<td>0.034</td>
<td>0.034</td>
<td>1</td>
</tr>
<tr>
<td>TR2</td>
<td>TR5</td>
<td>0.045</td>
<td>0.045</td>
<td>0.045</td>
<td>1</td>
</tr>
<tr>
<td>TR2</td>
<td>TR6</td>
<td>0.045</td>
<td>0.045</td>
<td>0.045</td>
<td>1</td>
</tr>
<tr>
<td>TR3</td>
<td>TR5</td>
<td>0.045</td>
<td>0.045</td>
<td>0.045</td>
<td>1</td>
</tr>
<tr>
<td>TR3</td>
<td>TR6</td>
<td>0.045</td>
<td>0.045</td>
<td>0.045</td>
<td>1</td>
</tr>
<tr>
<td>TR2</td>
<td>TR7</td>
<td>0.036</td>
<td>0.036</td>
<td>0.036</td>
<td>1</td>
</tr>
<tr>
<td>TR3</td>
<td>TR7</td>
<td>0.045</td>
<td>0.045</td>
<td>0.045</td>
<td>1</td>
</tr>
<tr>
<td>TR5</td>
<td>TR6</td>
<td>0.045</td>
<td>0.045</td>
<td>0.045</td>
<td>1</td>
</tr>
<tr>
<td>TR2</td>
<td>TR7B</td>
<td>0.139</td>
<td>0.139</td>
<td>0.139</td>
<td>1</td>
</tr>
<tr>
<td>TR3</td>
<td>TR7B</td>
<td>0.139</td>
<td>0.139</td>
<td>0.139</td>
<td>1</td>
</tr>
</tbody>
</table>

*** All connection combinations passed ***

Adjustment Statistics, Traverse 3

Statistics
Solution converged in 2 iterations
Total Observations 49
Total Relatives: 70
Degrees of Freedom: 33

Observation Count: 49
Degrees of Freedom: 33
Chi-Square Test: 0.804
Standard Error Unit Weight: 1.150
Passed the Chi-Square test at the 95.00 significance level
Traverse 4

- 2 fixed control points
- 8 occupations
- Each angle measured in direct & reverse, 4 sets
- Same standard errors as traverse 1

Adjusted Coordinate Error Ellipses

Traverse 4
95% confidence interval

<table>
<thead>
<tr>
<th>Sta.</th>
<th>Semi-Major Axis</th>
<th>Semi-Minor Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trav 1</td>
<td>Trav 4</td>
</tr>
<tr>
<td>TR2</td>
<td>0.087</td>
<td>0.018</td>
</tr>
<tr>
<td>TR1B</td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>TR3</td>
<td>0.111</td>
<td>0.026</td>
</tr>
<tr>
<td>TR4</td>
<td>0.135</td>
<td>0.034</td>
</tr>
<tr>
<td>TR5</td>
<td>0.114</td>
<td>0.027</td>
</tr>
<tr>
<td>TR6</td>
<td>0.107</td>
<td>0.028</td>
</tr>
<tr>
<td>TR7</td>
<td>0.072</td>
<td>0.017</td>
</tr>
<tr>
<td>TR7B</td>
<td>0.139</td>
<td>0.031</td>
</tr>
</tbody>
</table>

ALTA Tolerances, Traverse 4

ALTA CONNECTIVITY: Tolerance of 0.074 ± 60 ppm at the 95% CI
All possible connections between the following points were checked:
TR2, TR1B, TR3, TR5, TR6, TR7, TR7B, TR8, TR9, TR10
36 connections tested. All internal relative error ellipses will be shown.

<table>
<thead>
<tr>
<th>Sta.</th>
<th>Semi-Major Axis</th>
<th>Semi-Minor Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>Allowable</td>
</tr>
<tr>
<td>TR1B</td>
<td>0.074</td>
<td>0.074</td>
</tr>
<tr>
<td>TR1B</td>
<td>0.074</td>
<td>0.074</td>
</tr>
<tr>
<td>TR3</td>
<td>0.074</td>
<td>0.074</td>
</tr>
<tr>
<td>TR5</td>
<td>0.074</td>
<td>0.074</td>
</tr>
<tr>
<td>TR6</td>
<td>0.074</td>
<td>0.074</td>
</tr>
<tr>
<td>TR7</td>
<td>0.074</td>
<td>0.074</td>
</tr>
<tr>
<td>TR7B</td>
<td>0.074</td>
<td>0.074</td>
</tr>
<tr>
<td>TR8</td>
<td>0.074</td>
<td>0.074</td>
</tr>
<tr>
<td>TR9</td>
<td>0.074</td>
<td>0.074</td>
</tr>
<tr>
<td>TR10</td>
<td>0.074</td>
<td>0.074</td>
</tr>
</tbody>
</table>

*** All connections passed ***
Adjustment Statistics, Traverse 4

Statistics
********
Solution converged in 2 iterations.
Total Observations 85
Total Degrees of Freedom 69

<table>
<thead>
<tr>
<th>Observation Count</th>
<th>Sum Squares</th>
<th>Error Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angles</td>
<td>75</td>
<td>43.022</td>
</tr>
<tr>
<td>Distances</td>
<td>10</td>
<td>5.689</td>
</tr>
<tr>
<td>Total</td>
<td>85</td>
<td>48.601</td>
</tr>
</tbody>
</table>

Reference Variance: 0.704
Standard Error Unit Weight: (~)=0.839
Passed the Chi-Square test at the 95.00 confidence level
47.924 << 48.601 << 93.866

Traverse 5

- 2 fixed control points
- 10 occupations
- Each angle measured in direct & reverse, 4 sets
- Same standard errors as traverse 1

Adjusted Coordinate Error Ellipses

Traverse 5
95% confidence interval

<table>
<thead>
<tr>
<th>Sta.</th>
<th>Semi-Major Axis</th>
<th>Semi-Minor Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trav 4</td>
<td>Trav 5</td>
</tr>
<tr>
<td>TR2</td>
<td>0.018</td>
<td>0.023</td>
</tr>
<tr>
<td>TR1B</td>
<td>0.02</td>
<td>0.023</td>
</tr>
<tr>
<td>TR3</td>
<td>0.026</td>
<td>0.032</td>
</tr>
<tr>
<td>TR4</td>
<td>0.034</td>
<td>0.04</td>
</tr>
<tr>
<td>TR5</td>
<td>0.027</td>
<td>0.032</td>
</tr>
<tr>
<td>TR6</td>
<td>0.026</td>
<td>0.033</td>
</tr>
<tr>
<td>TR7</td>
<td>0.017</td>
<td>0.022</td>
</tr>
<tr>
<td>TR7B</td>
<td>0.031</td>
<td>0.023</td>
</tr>
</tbody>
</table>
**ALTA Tolerances, Traverse 5**

<table>
<thead>
<tr>
<th>No</th>
<th>Obs</th>
<th>X (ft)</th>
<th>Y (ft)</th>
<th>X Allowable Error</th>
<th>Y Allowable Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR2</td>
<td>TR1</td>
<td>140.412</td>
<td>0.015</td>
<td>0.015</td>
<td>0.020</td>
</tr>
<tr>
<td>TR1</td>
<td>TR2</td>
<td>155.471</td>
<td>0.014</td>
<td>0.014</td>
<td>0.020</td>
</tr>
<tr>
<td>TRB</td>
<td>TR6</td>
<td>150.254</td>
<td>0.018</td>
<td>0.018</td>
<td>0.026</td>
</tr>
<tr>
<td>TR6</td>
<td>TRB</td>
<td>179.122</td>
<td>0.027</td>
<td>0.027</td>
<td>0.039</td>
</tr>
</tbody>
</table>

**Adjustment Statistics, Traverse 5**

Statistics

Solution converged in 2 iterations.

Total Observations: 98
Total Traverse: 16
Degrees of Freedom: 82

Observation Count

<table>
<thead>
<tr>
<th>Type</th>
<th>Mean Squares</th>
<th>Error Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angles:</td>
<td>51.293</td>
<td>2.985</td>
</tr>
<tr>
<td>Distances:</td>
<td>60.214</td>
<td>2.985</td>
</tr>
</tbody>
</table>

Total: 102.497  1.117

Reference Variance: 1248

Standard Error Unit Weight: +/-1.117

Passed the Chi-Square test at the 95.00 significance level

18.845 < 162.297 < 188.327

---

**Traverse 6**

- 2 fixed control points
- Sideshots added to Traverse 5
Relative Positional Precision
Explained in Everyday Language

ALTA Tolerances, Traverse 6

Adjustment Statistics, Traverse 6

Traverse 6, Revised Errors

Traverse 6, Revised Errors

Allowable RPP improved with more realistic standard errors.

Demonstration Project 2

Combined Network
GNSS and Open Traverse

Project 2 Scope

- 4 instrument traverse occupations
- 8 GNSS vectors
Standard Errors, Project 2

- Distance
  - Constant: 0.0075m  Scalar: 2 PPM
- Horizontal angle
  - Pointing: 1.0"
- Vertical angle
  - Pointing: 3.0"

Standard Errors, Project 2

- Total Station
  - Centering: 0.002m  Height: 0.002m
- Target
  - Centering: 0.002m  Height: 0.002m
- GNSS errors
  - Factor supplied std errors by 8.0
  - Centering: 0.002m  Height: 0.002m

Allowable RPP Settings

Enter other tolerance values here.
Enter ALTA tolerance values here.
Allowable RPP Settings

Optical Measurements

- 4 instrument points
  - Open traverse

Two control points

Optical Measurement Data

Fixed control positions

Traverse measurements
GNSS Observations

- 8 GNSS vectors
  - 4 static sessions

The better the network, the more circular the error ellipses.
### Adjustment Statistical Summary

- **Iterations**: 3
- **Number of Stations**: 9
- **Number of Observations**: 68
- **Number of Unknowns**: 24
- **Number of Redundant Obs**: 24

<table>
<thead>
<tr>
<th>Observation</th>
<th>Count</th>
<th>Sum Squares</th>
<th>Error of Fit/Stdev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angles</td>
<td>8</td>
<td>9.944</td>
<td>1.156</td>
</tr>
<tr>
<td>Distances</td>
<td>8</td>
<td>1.236</td>
<td>1.029</td>
</tr>
<tr>
<td>Elevations</td>
<td>8</td>
<td>1.153</td>
<td>1.236</td>
</tr>
<tr>
<td>GPS Deltas</td>
<td>24</td>
<td>4.892</td>
<td>0.659</td>
</tr>
</tbody>
</table>

- The Chi-Square Test at 5.00% level passed Lower/Upper Bounds (0.714/1.201)

### Station Coordinate Standard Deviations (Meters)

<table>
<thead>
<tr>
<th>Station</th>
<th>N</th>
<th>E</th>
<th>Elev</th>
</tr>
</thead>
<tbody>
<tr>
<td>0012</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>0013</td>
<td>0.002664</td>
<td>0.002517</td>
<td>0.000000</td>
</tr>
<tr>
<td>0015</td>
<td>0.003059</td>
<td>0.002957</td>
<td>0.004816</td>
</tr>
<tr>
<td>0016</td>
<td>0.001954</td>
<td>0.001876</td>
<td>0.003328</td>
</tr>
<tr>
<td>0017</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>0018</td>
<td>0.002536</td>
<td>0.002478</td>
<td>0.004381</td>
</tr>
<tr>
<td>0051</td>
<td>0.006219</td>
<td>0.006053</td>
<td>0.023305</td>
</tr>
<tr>
<td>0052</td>
<td>0.006476</td>
<td>0.007042</td>
<td>0.019776</td>
</tr>
<tr>
<td>0053</td>
<td>0.007334</td>
<td>0.008010</td>
<td>0.023305</td>
</tr>
</tbody>
</table>

### Station Coordinate Error Ellipses (Meters)

- **Confidence Region = 95%**

<table>
<thead>
<tr>
<th>Station</th>
<th>Semi-Major Axis</th>
<th>Semi-Minor Axis</th>
<th>Azimuth of Major Axis</th>
<th>Elev</th>
</tr>
</thead>
<tbody>
<tr>
<td>0012</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>0013</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>0015</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>0016</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>0017</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>0018</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>0051</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>0052</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
<tr>
<td>0053</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td></td>
</tr>
</tbody>
</table>

---

Todd Horton, PE, PLS
Positional Tolerance Check

<table>
<thead>
<tr>
<th>Station</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Distance</th>
<th>Actual</th>
<th>Allowed</th>
<th>Basic</th>
<th>Allowance</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0001</td>
<td>43.2434</td>
<td>-88.7654</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>0002</td>
<td>43.2433</td>
<td>-88.7653</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>0003</td>
<td>43.2432</td>
<td>-88.7652</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>0004</td>
<td>43.2431</td>
<td>-88.7651</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Least Squares Adjustment

Interpreting Results

Coordinate Standard Deviations and Error Ellipses

- Coordinate standard deviations represent the accuracy of the coordinates.
- Error ellipses are a graphical representation of the standard deviations.
Coordinate Standard Deviations and Error Ellipses

- The better the network, the more circular the error ellipses.

- High standard deviations can be found in networks with a good standard deviation of unit weight and well weighted observations due to effects of the network geometry.

Degree of freedom = Redundancy

<table>
<thead>
<tr>
<th>Statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution converged in 2 iterations</td>
<td></td>
</tr>
<tr>
<td>Total Observations: 20</td>
<td></td>
</tr>
<tr>
<td>Total Unknowns: 16</td>
<td></td>
</tr>
<tr>
<td>Degrees of Freedom: 4</td>
<td></td>
</tr>
</tbody>
</table>

- The degree of freedom is an indication of how many redundant measurements are in the survey.

- Degree of freedom is defined as the number of measurements in excess of the number of measurements necessary to solve the network.

Standard Residual

- The standard residual is a measure of the similarity of the residual to the a-priori standard error.

\[
\text{StdRes} = \frac{\text{Residual}}{\text{StdErr}}
\]

- The standard residual is the measurement's residual divided by the standard error displayed in the unadjusted measurement section.
Relative Positional Precision
Explained in Everyday Language

Observed Distances

<table>
<thead>
<tr>
<th>From Sta</th>
<th>To Sta</th>
<th>Dist</th>
<th>StErr</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR1</td>
<td>TR100</td>
<td>820.991</td>
<td>0.011</td>
</tr>
<tr>
<td>TR1</td>
<td>TR2</td>
<td>867.486</td>
<td>0.011</td>
</tr>
<tr>
<td>TR1</td>
<td>TR1B</td>
<td>218.387</td>
<td>0.01</td>
</tr>
<tr>
<td>TR2</td>
<td>TR3</td>
<td>804.571</td>
<td>0.011</td>
</tr>
<tr>
<td>TR3</td>
<td>TR4</td>
<td>491.926</td>
<td>0.009</td>
</tr>
<tr>
<td>TR4</td>
<td>TR5</td>
<td>871.245</td>
<td>0.011</td>
</tr>
<tr>
<td>TR5</td>
<td>TR6</td>
<td>823.002</td>
<td>0.011</td>
</tr>
<tr>
<td>TR6</td>
<td>TR7</td>
<td>485.811</td>
<td>0.01</td>
</tr>
<tr>
<td>TR7</td>
<td>TR7B</td>
<td>821.034</td>
<td>0.016</td>
</tr>
<tr>
<td>TR7</td>
<td>TR100</td>
<td>808.408</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Adjusted Distances

<table>
<thead>
<tr>
<th>From Sta</th>
<th>To Sta</th>
<th>Distance</th>
<th>Residual</th>
<th>StdRes</th>
</tr>
</thead>
<tbody>
<tr>
<td>TR1</td>
<td>TR100</td>
<td>820.99</td>
<td>-0.001</td>
<td>0.1</td>
</tr>
<tr>
<td>TR1</td>
<td>TR2</td>
<td>867.491</td>
<td>0.004</td>
<td>0.4</td>
</tr>
<tr>
<td>TR1</td>
<td>TR1B</td>
<td>218.387</td>
<td>0.000</td>
<td>0.0</td>
</tr>
<tr>
<td>TR2</td>
<td>TR3</td>
<td>804.57</td>
<td>0.000</td>
<td>0.0</td>
</tr>
<tr>
<td>TR3</td>
<td>TR4</td>
<td>491.926</td>
<td>0.000</td>
<td>0.0</td>
</tr>
<tr>
<td>TR4</td>
<td>TR5</td>
<td>871.24</td>
<td>-0.004</td>
<td>0.4</td>
</tr>
<tr>
<td>TR5</td>
<td>TR6</td>
<td>822.998</td>
<td>-0.004</td>
<td>0.4</td>
</tr>
<tr>
<td>TR6</td>
<td>TR7</td>
<td>485.811</td>
<td>0.000</td>
<td>0.0</td>
</tr>
<tr>
<td>TR7</td>
<td>TR7B</td>
<td>820.509</td>
<td>0.000</td>
<td>0.0</td>
</tr>
<tr>
<td>TR7</td>
<td>TR100</td>
<td>808.408</td>
<td>0.000</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Standard Residual

• A standard residual greater than 2 is typically flagged.

• A high standard residual may be an indication of a blunder.

• A large number of high standard residuals may indicate that the entered standard errors are not realistic.
Wisconsin Society of Land Surveyors

Relative Positional Precision Explained in Everyday Language

Total Observations: 20
Total Unknowns: 16
Degrees of Freedom: 4

<table>
<thead>
<tr>
<th>Observation Type</th>
<th>Count</th>
<th>Sum Squares of Std Res</th>
<th>Error Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angles</td>
<td>10</td>
<td>3.648</td>
<td>1.351</td>
</tr>
<tr>
<td>Distances</td>
<td>10</td>
<td>0.444</td>
<td>0.471</td>
</tr>
<tr>
<td>Total</td>
<td>20</td>
<td>4.092</td>
<td>1.011</td>
</tr>
</tbody>
</table>

Reference Variance: 1.023
Standard Error Unit Weight: (+/-)1.011
Passed the Chi-Square test at the 95.00 significance level
0.484 <= 4.092 <= 11.143

Error Factor

\[ \text{ErrorFactor}_{\text{Type}} = \sqrt{\sum (\text{StdRes}_{\text{Type}})^2 / r \cdot n / k} \]

- \( r \) = degrees of freedom (redundancy)
- \( n \) = total number of observations
- \( k \) = number of observations of Type

<table>
<thead>
<tr>
<th>Observation Type</th>
<th>Count</th>
<th>Sum Squares of Std Res</th>
<th>Error Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distances</td>
<td>10</td>
<td>0.444</td>
<td>0.471</td>
</tr>
</tbody>
</table>

Error Factor

- Error Factors should be roughly equal, and should all approximately be within a range of 0.5 to 1.5.
- If for example, the Error Factor for angles is equal to 15.7 and that for distances is equal to 2.3, then there is very likely a problem with the angles.
Total Error Factor

\[ \text{Total Error Factor} = \sqrt{\sum (\text{StdRes})^2} / r = S_o \]

- \( r = \) degrees of freedom (redundancy)

<table>
<thead>
<tr>
<th>Observation Count</th>
<th>Sum Squares of Std Res</th>
<th>Error Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angles: 10</td>
<td>3.648</td>
<td>1.351</td>
</tr>
<tr>
<td>Distances: 10</td>
<td>0.444</td>
<td>0.471</td>
</tr>
<tr>
<td>Total: 20</td>
<td>4.092</td>
<td>1.011</td>
</tr>
</tbody>
</table>

Total Error Factor = Standard Error Of Unit Weight

Standard Error Of Unit Weight

- The standard error of unit weight relates to the overall adjustment and not to an individual measurement.

\(- S_o = 1\) indicates that the results of the adjustment are consistent with the a-priori standard errors.

Reference Variance

- The Reference Variance is the Standard Error of Unit Weight squared.

<table>
<thead>
<tr>
<th>Observation Count</th>
<th>Sum Squares of Std Res</th>
<th>Error Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angles: 10</td>
<td>3.648</td>
<td>1.351</td>
</tr>
<tr>
<td>Distances: 10</td>
<td>0.444</td>
<td>0.471</td>
</tr>
<tr>
<td>Total: 20</td>
<td>4.092</td>
<td>1.011</td>
</tr>
</tbody>
</table>

Reference Variance: 1.023
Standard Error Unit Weight: (+/-)1.011
Chi-square Test

- The chi-square test result describes the quality of fit of the adjustment.
  - The a-priori standard errors are used to determine the weights of the measurements.
  - These standard errors can also be looked at as an estimate of how accurately the measurements were made.

Chi-square Test

- The chi-square distribution is a function of degrees of freedom.

<table>
<thead>
<tr>
<th>Degrees of Freedom:</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation Count</td>
<td>20</td>
</tr>
<tr>
<td>Sum Squares of Std Res</td>
<td>4.092</td>
</tr>
<tr>
<td>Error Factor</td>
<td>1.011</td>
</tr>
</tbody>
</table>

Passed the Chi-Square test at the 95.00 significance level

\[ 0.484 \leq 4.092 \leq 11.143 \]

Significance Level

- Significance ≠ Confidence

- Typical significance for Chi-square test is 95%.
  - 95% chance of result being true, and
    - Used in Carlson Surv Net
  - 5% chance of result being false
    - Used in Microsurvey Star*Net
Chi Square & Degrees of Freedom

<table>
<thead>
<tr>
<th>DoF</th>
<th>$X^2_{va}$</th>
<th>$X^2_{vb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.001</td>
<td>5.02</td>
</tr>
<tr>
<td>2</td>
<td>0.051</td>
<td>7.38</td>
</tr>
<tr>
<td>3</td>
<td>0.216</td>
<td>9.35</td>
</tr>
<tr>
<td>4</td>
<td>0.484</td>
<td>11.1</td>
</tr>
<tr>
<td>5</td>
<td>0.831</td>
<td>12.8</td>
</tr>
<tr>
<td>6</td>
<td>1.24</td>
<td>14.4</td>
</tr>
<tr>
<td>7</td>
<td>1.69</td>
<td>16.0</td>
</tr>
<tr>
<td>8</td>
<td>2.18</td>
<td>17.5</td>
</tr>
<tr>
<td>9</td>
<td>2.70</td>
<td>19.0</td>
</tr>
<tr>
<td>10</td>
<td>3.25</td>
<td>20.5</td>
</tr>
<tr>
<td>12</td>
<td>4.40</td>
<td>23.3</td>
</tr>
<tr>
<td>14</td>
<td>5.63</td>
<td>26.1</td>
</tr>
<tr>
<td>16</td>
<td>6.91</td>
<td>28.8</td>
</tr>
<tr>
<td>18</td>
<td>8.23</td>
<td>31.5</td>
</tr>
<tr>
<td>20</td>
<td>9.59</td>
<td>34.2</td>
</tr>
<tr>
<td>25</td>
<td>13.1</td>
<td>40.6</td>
</tr>
<tr>
<td>30</td>
<td>16.8</td>
<td>47.0</td>
</tr>
<tr>
<td>40</td>
<td>24.4</td>
<td>59.3</td>
</tr>
<tr>
<td>60</td>
<td>40.5</td>
<td>83.3</td>
</tr>
</tbody>
</table>

Chi-square Test

- The chi-square test merely tests whether the results of the adjusted measurements are consistent with the a-priori standard errors.
- If you change the project standard errors and then reprocess the survey the results of the chi-square test change, even though the measurements themselves did not change.

Failing on the low end

Failing on the low end indicates that the data is actually better than expected compared to our a-priori standard errors.

Degrees of Freedom: 84
Reference Variance: 0.657
Standard Error Unit Weight: (+/-)0.810
Failed the Chi-Square test at the 95.00 significance level

$60.540 \leq 55.171 \leq 111.242$
Failing on the high end

If the test failed on the high end, then check for:

- blunders in your actual observations,
- field book recording errors, or
- data preparation errors such as:
  - incorrectly entered measurements or
  - misnamed stations in the input data file.

Reporting Compliance with Standards

Allowable RPP

0.07 feet + 50 ppm
**ALTA allowable RPP**

- What is the allowable ALTA RPP for a line of 1,000 feet?
  - Constant component = 0.07 ft
  - Scalar component = 50 ppm
    - $1000 \times \left(\frac{50}{1,000,000}\right) = 0.05$ feet

- RPP = $0.07 + 0.05 = 0.12$ ft

*Note: the constant and scalar components are not random errors, so the error of a sum does not apply.*

**WI ADMIN CODE, Chapter A−E 7**
**MINIMUM STANDARDS FOR PROPERTY SURVEYS**

A−E 7.06 Measurements.

(3) The maximum allowable deviation in relative positional accuracy for a survey is plus or minus 0.07 foot plus 50 parts per million, based on the direct distance between the two corners being tested.

**ALTA Measurement Standards**

It is recognized that in certain circumstances, the size or configuration of the surveyed property, or the relief, vegetation or improvements on the surveyed property will result in survey measurements for which the maximum allowable Relative Positional Precision may be exceeded.

If the maximum allowable Relative Positional Precision is exceeded, the surveyor shall note the reason as explained in Section 6.B.ix below.
Allowable RPP Settings

This dialog box allows the user to define the points to be included in the ALTA report processing.

Traverse 6, Revised Errors

Allowable RPP improved with more realistic standard errors.
Planning for good RPP

1. Understand the primary sources of random errors in your measurements.

2. Develop reliable estimates of the random error sizes from each of those sources.

3. Use a correctly weighted Least Squares adjustment to process your data.

Contact Information

Todd Horton, PE, PLS

Todd.Horton.PE.PLS@gmail.com
(217) 493-3371 mobile

thorton@parkland.edu
(217) 373-3785