# MULTIPLE COMPONENT ANALYSIS OF FIBER LENGTH DISTRIBUTIONS

Gerard J. F. Ring, Associate Professor, UW-SP Department of Paper Science
Aric J. Bacon, Student, UW-SP Department of Paper Science

ABSTRACT

Blending two or more pulps in a papermaking furnish is an important method of obtaining desirable paper properties by taking advantage of selected fiber properties. However, measuring the percent composition of individual pulps in a blended furnish generally means using tedious fiber counting techniques. An alternative technique for two-component furnishes using an optical fiber-length analyzer is also available that uses an empirical calibration to relate percent composition to the length-weighted fiber-length average of the resultant pulp blend.

This paper presents a more general mathematical technique for expressing the fiber-length averages of distributions obtained from optical fiber-length analyzers. This technique permits the derivation of accurate mathematical relationships between pulp component percentages and all fiber length averages, whether arithmetic, length-weighted or weight-weighted. For example, the derived relationship for the length-weighted average ( $\overline{L}_{blend}$ ) of a hardwood/softwood blend is

$$\overline{L}_{blend} = \frac{\dfrac{\overline{L}_H x_H}{Q_H} + \dfrac{\overline{L}_S \left(1 - x_H\right)}{Q_S}}{\dfrac{x_H}{Q_H} + \dfrac{\left(1 - x_H\right)}{Q_S}}$$

where $x_H$ is the percent hardwood in the blend, Q is the coarseness for each pulp and $\overline{L}$ is the length-weighted fiber length average for each pulp in the furnish.

The derived relations are not limited to two-pulp furnishes but can be used with three or more components. A practical application of this technique allows calculation of instantaneous pulp component percentages of rapidly changing pulp blends with on-line fiber length analyzers.

## INTRODUCTION

Optical fiber-length analyzers tend to be viewed only as expensive alternatives to classification. In reality, these new instruments far surpass the limited capabilities of classifiers because they provide papermakers with the means to study detailed and subtle changes in the shape of fiber length distributions that occur with processing. They also allow papermakers to make accurate correlations between paper properties and fiber-length distributions. However, because optical analyzers are relatively new, the techniques that exploit the potential of these instruments have not been developed.

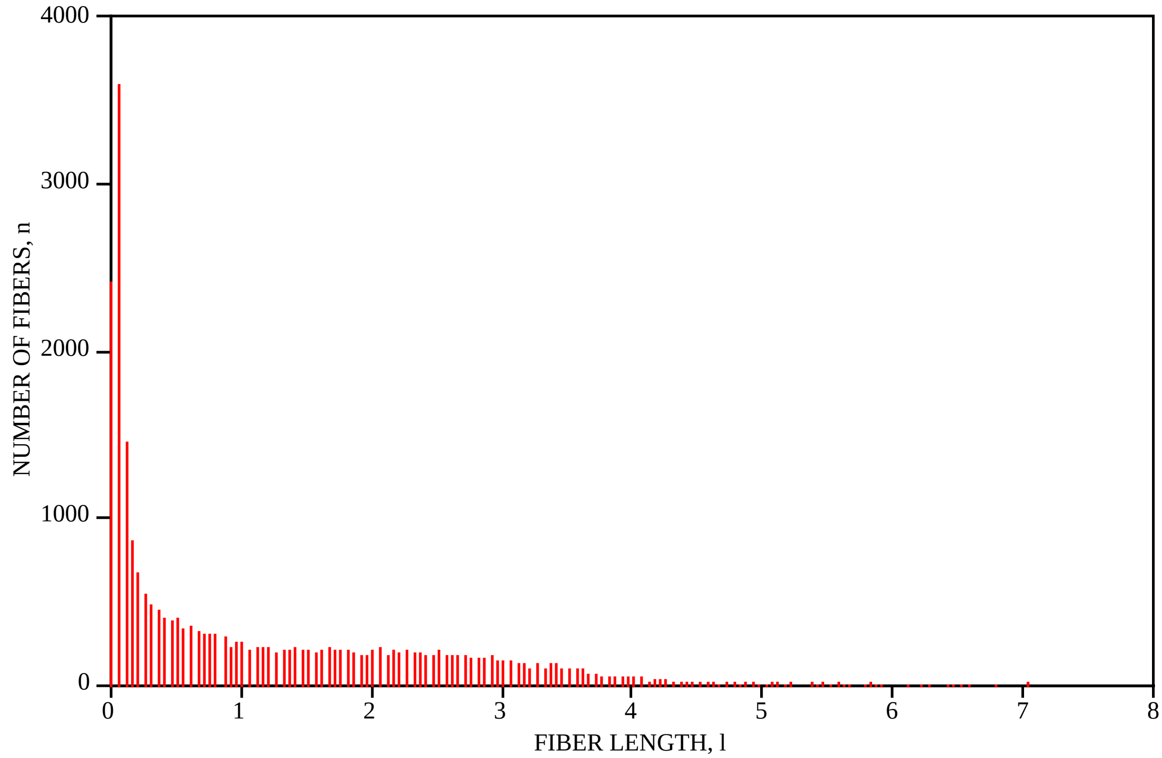
This paper presents a mathematical technique to determine the weight percent composition of multiple pulp components in a mixed or blended pulp. The technique is based on the use of distribution moments to measure the shape of fiber-length distributions. Moments are not a new idea to fiber length analysis, since they are fundamental to the definition of fiber length averages. The derived equations are precise analytic descriptions of the fiber length distributions and may be used to determine the accuracy and precision of measuring instruments.

## PULP FIBER-LENGTH DISTRIBUTIONS

The length of unpulped wood fibers varies for many reasons. For a given species, both genetics and the environment will be influential factors, while in a given tree, age and specific fiber location will also contribute to fiber length variations (7). A relatively narrow distribution will result when the fiber length of unpulped wood is measured and its average value will be characteristic of the particular tree species. For example, hardwood species have average fiber lengths about 1 mm while softwood species have fiber lengths usually more than 3 mm (4). Specific examples of hardwoods include: sugar maple, 1.0 mm; silver birch, 1.2 mm and aspen, .9 mm. Specific examples of softwoods include: jack pine, 3.5 mm; ponderosa pine, 3.6 mm and redwood, 7.0 mm.

The initial, in-situ fiber-length distribution will define the maximum fiber length that can be derived from pulp manufactured from the wood. The effect of processing, mainly pulping and refining, will shorten many of the fibers before they are consolidated into a sheet of paper. Not only will fibers be broken or cut, but other shorter wood elements such as ray and parenchyma cells will be liberated; furthermore, the fibrillar layers of the cell wall

Figure 1. An example of a raw-data, fiber-length distribution for a kraft softwood.




may become unraveled. The cellular debris produced by pulping is referred to as primary fines. The cellular debris produced by refining, is referred to as secondary fines and may be microfibrillar strands of considerable length (6). Thus, any pulp fiber-length distribution that is observed after pulping and/or papermaking is a function of both the original fiber length distribution and the effects of processing.

Because fiber length distributions affect tensile strength, tear, opacity, porosity and many other paper properties, papermakers have developed many techniques to measure fiber length averages (3). Three techniques are described by TAPPI test procedures: Fiber Length of Pulp by Projection (T 232), Fiber Length of Pulp by Classification (T 233), and Fiber Length of Pulp and Paper by Automated Optical Analyzer (T 271). The two older procedures, which have now been designated as classical methods, have been outdated by the development of automated optical analyzers. These new instruments provide a greater quantity of data, more precise fiber-length measurements and a complete depiction of the fiber-length distributions.

## FIBER LENGTH MOMENTS AND DISTRIBUTIONS

Optical fiber length analyzers count the number of fibers that produce a discrete, digital response from their measuring sensors. In the case of the Kajaani FS-200 (described by TAPPI T 271), the length of a single fiber is related to the number of adjacent optical diodes that are interrupted from the impingement of a polarized laser beam when the fiber passes through a glass capillary situated between the beam and the diodes. A total of 144 diodes are linearly arrayed down the length of the capillary. Each diode is .05 mm in length for a total sensing span of 7.2 mm.

After each analysis involving the measurement of thousands of fibers, the FS-200 produces 144 pieces of raw data. Each piece of data corresponds to the number of fibers detected by the diode array as having a classified length equal to the number of interrupted diodes multiplied by .05 mm. The raw data forms a frequency distribution or histogram of fiber count (n) versus fiber length (l). An example of a typical raw-data histogram for an unrefined Kraft softwood pulp is depicted in Figure 1.

The shape of a fiber-length distribution is extremely important because it holds information about the pulp's original fiber distribution and its processing history. The fiber-length frequency distribution for the pulp in Figure 1 is clearly not normal or gaussian in shape, but is distinctly skewed towards the lowest fiber lengths. This severe skewing is a result of fiber fragmentation during pulping. The original distribution (which was possibly non-skewed) was expanded in the number of shorter fiber lengths as each fiber broke into two shorter segments. Monitoring the shape of pulp fiber-length distributions as a function of processing holds great potential for quality control because fiber-length distributions are not just measures of fiber length but also measures of fines content. Controlling a process to a single fiber-length average may produce erroneous results because two pulp samples with identical fiber-length averages may have different ratios of fines and long fibers.

The shapes of distributions may be precisely monitored by calculating their moments as defined by the expression

$$m'_k = \frac{\sum_{i=0}^{h} n_i l_i^k}{\sum_{i=0}^{h} n} = \frac{\sum_{i=0}^{h} n_i l_i^k}{N} \tag{1}$$

where $N$ is the total number of fibers, $h$ is the total number of fiber-length classes, and $n$ is the total number of fibers in any fiber-length class '$i$' described by class mid-point '$l_i$' (5). The parameter "$k$" is an integer referring to the number of the moment. There can be an infinite number of moments describing a single distribution, but the number of moments should never exceed the number of classes. Since, the objective of using moments is to summarize the distribution, it is pointless to use more than is necessary (usually one or two). Equation (1) describes the "$k$ th" moment about the origin. The average or mean (μ) of a distribution is the first moment about the origin ($k$=1; $\mu = m'_1$). The zeroth moment is trivial and always equal to unity.

In essence, the moments of a distribution describe its position on the fiber length axis (x-axis). The first moment (the mean or average) describes the central position while higher moments (the second, third etc.) describe the dispersion of the distribution. The terms dispersion and shape are synonymous in this discussion. Usually, two distributions differ by their central position and the first moment is adequate to describe differences between the two. However, in similar distributions where only shape varies, the second moments will be required. In more extreme situations where only subtle variations in the distributional shapes occur, the third or higher moments will be required to distinguish the two. In theory, any two differing distributions will eventually be differentiated by utilizing a sufficient number of moments.

## FIBER LENGTH AVERAGES

Fiber length averages have been used in the paper industry to characterize fiber length distributions prior to the advent of optical fiber-length analyzers. There are three types of averages in use today: the arithmetic and two weighted averages. The arithmetic or number average is simply the mean of a fiber length distribution and is equal to the first moment as depicted by the equation below:

$$\overline{A} = m'_1 = \frac{\sum nl}{\sum n} = \frac{\sum nl}{N}\,. \tag{2}$$

As mentioned before, this relationship describes the central position of the distribution depicted in Figure 1.

Weighted fiber-length averages use the fiber mass or weight instead of the fiber count as the numerical quantifier. This technique is necessary when an object's mass rather than its quantity governs a measured physical property. Molecular weight averages of polymers are examples—the number average MW correlates with colligative properties while the weight-average MW correlates with strength or viscosity properties. In the same manner, weighted averages have been preferred for pulp measurement because of the clear relationship between fiber length and sheet strength (3). However, since no colligative fiber properties exist, the arithmetic average has been prematurely judged unsuitable for most pulp characterizations.

The two weighted, fiber-length averages are defined for constant fiber coarseness (Q, the mass of the fiber per unit length {g/m}):

$$\overline{L} = \frac{\sum wl}{\sum w} = \frac{\sum (nQl)l}{\sum nQl} = \frac{Q\sum nl^2}{Q\sum nl} = \frac{\sum nl^2}{\sum nl} \tag{3}$$

and for non-constant fiber coarseness

$$\overline{W} = \frac{\sum wl}{\sum w} = \frac{\sum (nQl)l}{\sum nQl} = \frac{\alpha\sum nl^3}{\alpha\sum nl^2} = \frac{\sum nl^3}{\sum nl^2}\,. \tag{4}$$

In both equations, $w$ is the mass of $n$ fibers having coarseness $Q$ and length $l$. In Eq.(4), Clark's assumption that coarseness is linearly related to fiber length ($Q = \alpha\ l$) is utilized where $\alpha$ is an empirical factor of proportionality (3). Both weighted averages have calculated values greater than the arithmetic average and the values calculated from Eq. (4) exceed those calculated from Eq. (3). Weighting mathematically diminishes the presence of the smaller fibers in the final average. Thus, Eq. (4) emphasizes larger fibers more than Eq. (3) which emphasizes larger fibers more than the arithmetic average.

Clark maintained that Eq. (4) was the more suitable form for the weighted average fiber length because of his belief that shorter fibers were always proportionately less coarse (2,3). This belief probably has some merit for fibers that have never been processed and remain in the wood. However, fibers that are processed and undergo fragmentation may not have the simultaneous change in coarseness implicit in Clark's assumption for Eq. (4). A proportional change in coarseness of this nature corresponds to a longitudinal splitting of the pulp fiber during fragmentation. This phenomenon rarely happens because pulp fibers have cell walls that are composed of helical wrapped cellulose layers, and therefore tend to resist such fracturing. For this reason, Eq. (3), the weight average by length, has been accepted as a more appropriate fiber-length average for predicting pulp performance in the final sheet (1). From an historical perspective, the weight-weighted fiber length average was preferred because it produced a reasonable agreement between microscopic measurements (TAPPI T 232) and values calculated from classification results (TAPPI T233). Classifiers separate fibers not only according to length but by coarseness. Thus, Clark's assumption of a proportional relationship between fiber length and coarseness in Eq. (4) is appropriate in its application to classifiers.

Because of the accuracy of the distributional data obtained from optical fiber-length analyzers, it is no longer necessary or appropriate to use only one average to describe the fiber-length distribution. It is more appropriate to use all three fiber-length averages as distributional descriptors since they are functions of the distributional moments. The following redefinitions of each fiber length average demonstrate that the three fiber-length averages are simply the ratios of the distribution moments:

$$\overline{A} = \frac{\sum nl}{\sum n} = \frac{\sum nl}{\sum n} \times \frac{\sum n}{\sum n} = \frac{m'_1}{m'_0} = m'_1 \qquad (5)$$

$$\overline{L} = \frac{\sum nl^2}{\sum nl} = \frac{\sum nl^2}{\sum n} \times \frac{\sum n}{\sum nl} = \frac{m'_2}{m'_1} \qquad (6)$$

$$\overline{W} = \frac{\sum nl^3}{\sum nl^2} = \frac{\sum nl^3}{\sum n} \times \frac{\sum n}{\sum nl^2} = \frac{m'_3}{m'_2} \qquad (7)$$

By measuring all three fiber-length averages the shape of a fiber length distribution is monitored, and changes in furnish and processing can be carefully controlled. The remainder of this paper will develop Eqs. (5)-(7) into the necessary relationships to monitor furnish changes in a papermaking process.

## MULTIPLE COMPONENT FIBER LENGTH AVERAGES

Optical fiber analyzers 'see' an individual fiber passing through their sensors and measure its length. They are not able to discern the identity or source of a fiber and thus cannot distinguish the weight percentage of separate components in a mixed pulp. However, analysis of the resulting data can yield this information if the original distribution of each component is known. An empirical correlation procedure for measuring the percent composition of a hardwood/softwood blend utilizing the FS-200 has been incorporated into the software for this instrument. The following mathematical development will demonstrate a precise calculation of weight percentage in blended pulps of two or more components.

The total measured weight ($\hat{w}$) of a pulp can be expressed as the product of its coarseness ($Q$) and the summation of the length of every fiber

$$\hat{w} = Q \sum nl\ . \qquad (8)$$

Combining Equation (8) with Equations (2), (3), and (4) the following definitions are derived for all the summation terms:

$$\sum n = \hat{w} / \overline{A} Q \qquad (9)$$

$$\sum nl = \overline{A} \sum n = \hat{w} / Q \qquad (10)$$

$$\sum nl^2 = \overline{L} \sum nl = \overline{L}\, \hat{w} / Q \qquad (11)$$

$$\sum nl^3 = \overline{W} \sum nl^2 = \overline{WL}\, \hat{w} / Q \qquad (12)$$

The above definitions can then be substituted into the fiber-length average expression for pulps having '*j*' components.

$$\bar{A}_{mix} = \frac{\sum_j \left( \sum_i nl \right)}{\sum_j \left( \sum_i n \right)} = \frac{\sum_j \frac{\hat{w}_j}{Q_j}}{\sum_j \frac{\hat{w}_j}{\bar{A}_j Q_j}}, \tag{13}$$

$$\bar{L}_{mix} = \frac{\sum_j \left( \sum_i nl^2 \right)}{\sum_j \left( \sum_i nl \right)} = \frac{\sum_j \frac{\bar{L}_j \hat{w}_j}{Q_j}}{\sum_j \frac{\hat{w}_j}{Q_j}}, \tag{14}$$

and

$$\bar{W}_{mix} = \frac{\sum_j \left( \sum_i nl^3 \right)}{\sum_j \left( \sum_i nl^2 \right)} = \frac{\sum_j \frac{\bar{W}_j \bar{L}_j \hat{w}_j}{Q_j}}{\sum_j \frac{\bar{L}_j \hat{w}_j}{Q_j}}. \tag{15}$$

The combined coarseness for the pulp mixture or blend is defined as follows:

$$Q_{mix} = \frac{\sum_j \hat{w}_j}{\bar{A}_{mix}} = \frac{\sum_j \hat{w}_j}{\sum_j \frac{\hat{w}_j}{Q_j}}. \tag{16}$$

EXPERIMENTAL

The four pulps used in this analysis included a bleached softwood kraft, a bleached hardwood kraft, a thermo-mechanical pulp and a stone groundwood. These pulps were chosen to represent differences in both fiber length and coarseness.

All pulps were initially stored in a humidity and temperature controlled environment (50% RH, 23°C). Oven-dried weights for all pulps were determined prior to mixing. The procedures of TAPPI T 210 and T 271 were followed for all procedures.

All calculations of fiber length moments and averages were performed to four decimal places utilizing downloaded data from the Kajaani FS-200 and a spreadsheet especially designed for these calculations.

HARDWOOD/SOFTWOOD BLENDS

Fiber-length distributions, as depicted in Figure 1, are plots of 'n' versus 'l' where the

Figure 2. An example of an 'nl', weighted fiber-count distribution.

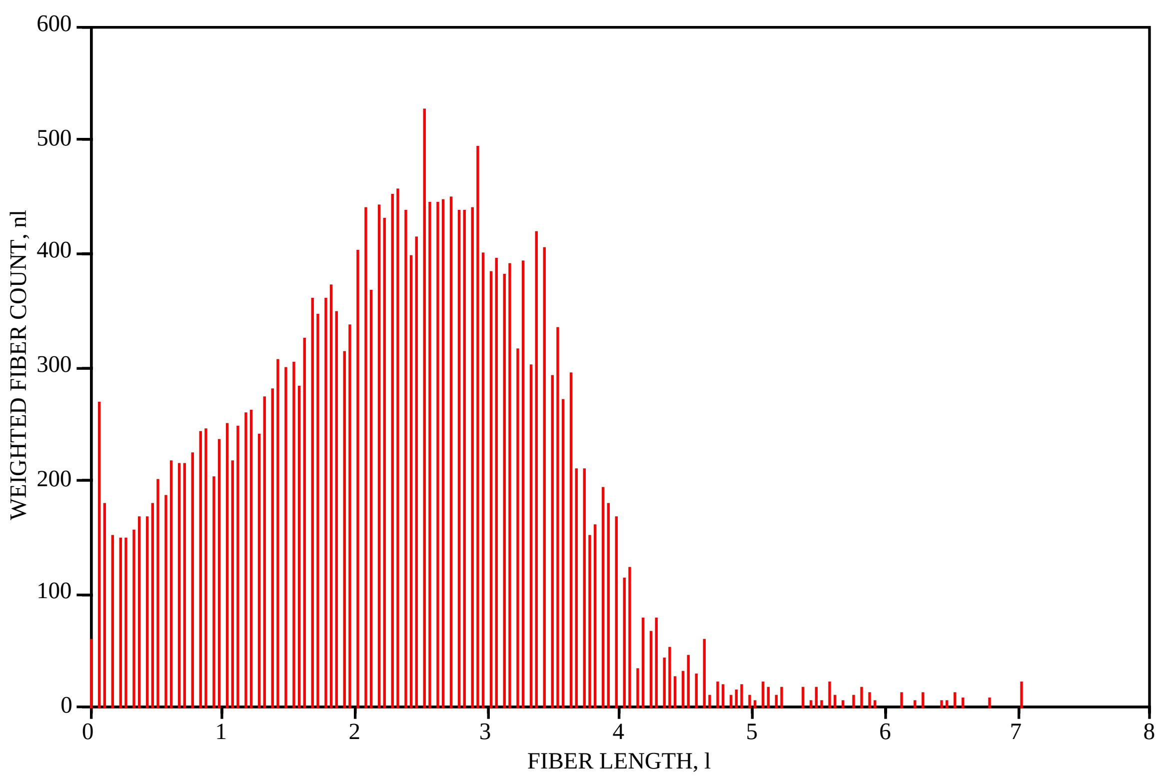


Table I. Fiber length averages for HW/SW blends.

| Fiber Length Average | Eq. |
|---|---|
| $\overline{A}_{mix} = \dfrac{\dfrac{x_H}{Q_H} + \dfrac{1 - x_H}{Q_S}}{\dfrac{x_H}{\overline{A}_H\, Q_H} + \dfrac{1 - x_H}{\overline{A}_S\, Q_S}}$ | (13) |
| $\overline{L}_{mix} = \dfrac{\dfrac{\overline{L}_H\, x_H}{Q_H} + \dfrac{\overline{L}_S\left(1 - x_H\right)}{Q_S}}{\dfrac{x_H}{Q_H} + \dfrac{\left(1 - x_H\right)}{Q_S}}$ | (14) |
| $\overline{W}_{mix} = \dfrac{\dfrac{\overline{W}_H\, \overline{L}_H\, x_H}{Q_H} + \dfrac{\overline{W}_S\, \overline{L}_S\left(1 - x_H\right)}{Q_S}}{\dfrac{\overline{L}_H\, x_H}{Q_H} + \dfrac{\overline{L}_S\left(1 - x_H\right)}{Q_S}}$ | (15) |

summative area of the histogram represents the total number of fibers that have been counted in an analysis. Quantitative analysis of weight percent composition requires the weighted distribution of 'n l' versus 'l', as depicted in Figure 2 which is generated from the same raw data as in Figure 1. The summative area of this histogram is directly related to the weight of the pulp sample, $\hat{w}$, through its measured coarseness as expressed in equation (8).

When two pulps are mixed together it follows that the total weight of this mixture is expressed as

$$\hat{w}_{mix} = Q_1 \sum n_1\, l + Q_2 \sum n_2\, l = \sum_j \left(Q_j \sum n_j\, l\right) . \quad (17)$$

Using the actual sample weight is difficult and seldom required in distributional analysis. It is more convenient to normalize the histogram or distribution area to unity or some other convenient value. Thus, for computation simplicity, the area of the 'n vs. l' distribution (Figure 1) for the pulp mixture will be normalized to unity producing a normalized weight, $w_{mix}$, or area of the 'nl versus l' distribution (Figure 2) equal to

$$w_{mix} = \frac{\hat{w}_{mix}}{\sum_j n_j} = \frac{\sum_j \left(Q_j \sum n_j\, l\right)}{\sum_j n_j} = Q_{mix}\, \overline{A}_{mix} . \quad (18)$$

This relationship then permits the expression of any single component weight, $w_j$, as a fraction, $x_j$, of the normalized total weight where $\sum x_j = 1$

$$w_j = \overline{A}_{mix}\, Q_{mix}\, x_j . \quad (19)$$

For blends of hardwood and softwood pulps, Eqs. (13)-(15) have been algebraically rewritten and presented in Table I with the letters H and S replacing the numeric 'j' subscripts.

Figure 3 plots % HW versus measured fiber-length average for all three average types for a series of HW/SW pulps that were accurately and precisely blended to a tenth of a milligram on a gravimetric balance. The average data is presented in Table II. Also plotted in Figure 3 are the theoretical curves of the fiber-length average equations from Table I. The data can be seen to fit the developed expressions very well. Correlating the calculated fiber length averages with their experimental values, the standard error of the estimates were determined to be 1.7%, 1.2%, and 1.4% for the arithmetic, length-weighted and weight-weighted averages respectively. The excellent agreement between the theoretical and the experimental values is significant because it means that the instrument used to generate the experimental values was accurately measuring the fiber length distributions and was not arbitrarily foreshortening long fibers or ignoring either short fibers or fines. This result should not be assumed to be obtainable from other optical fiber analyzers that use measuring principles different from that described by TAPPI T-271. Preliminary unpublished results from this study

Table II. Average data from hardwood and softwood blending experiment.

| % HW | A, mm | | L, mm | | W, mm | | Q, g/m | |
|---|---|---|---|---|---|---|---|---|
| | (exp) | (calc) | (exp) | (calc) | (exp) | (calc) | (exp) | (calc) |
| 0.0 | 1.120 | 1.130 | 2.288 | 2.303 | 2.811 | 2.827 | 0.198 | 0.176 |
| 9.9 | 0.969 | 0.938 | 2.148 | 2.089 | 2.786 | 2.731 | 0.136 | 0.166 |
| 19.3 | 0.847 | 0.820 | 1.972 | 1.908 | 2.683 | 2.633 | 0.136 | 0.157 |
| 30.0 | 0.732 | 0.727 | 1.759 | 1.723 | 2.532 | 2.511 | 0.128 | 0.148 |
| 40.0 | 0.681 | 0.664 | 1.621 | 1.569 | 2.431 | 2.388 | 0.128 | 0.140 |
| 49.8 | 0.617 | 0.617 | 1.447 | 1.434 | 2.257 | 2.258 | 0.121 | 0.134 |
| 60.3 | 0.565 | 0.577 | 1.306 | 1.301 | 2.100 | 2.104 | 0.123 | 0.127 |
| 68.9 | 0.537 | 0.550 | 1.207 | 1.202 | 1.973 | 1.967 | 0.144 | 0.123 |
| 81.1 | 0.493 | 0.519 | 1.059 | 1.073 | 1.733 | 1.752 | 0.114 | 0.116 |
| 91.0 | 0.493 | 0.498 | 0.988 | 0.978 | 1.596 | 1.555 | 0.109 | 0.112 |
| 100.0 | 0.482 | 0.482 | 0.897 | 0.897 | 1.357 | 1.358 | 0.108 | 0.108 |

demonstrated a disturbing "fines exclusion" effect in other instruments. These results will be published in a future paper.

## MULTIPLE COMPONENT BLENDS

Equations (13), (14), and (15) will predict a blended pulp's fiber-length averages for any number of individual pulp components. Table VI presents the average data for a series of experiments that mixed four separate pulps into various combinations of percentage blends. Table III presents the data for the four separate pulps used in this experiment that included two bleached chemical pulps and two unbleached mechanical pulps. Figure 4 presents the combined calculated data compared with the measured data for all three averages. Again very good agreement was found between the predicted values and the measured values. Correlating the predicted values with the measured values in Figure 4 gave standard errors for the estimated values

Figure 3. HW/SW blended pulp data with theoretical curve fitting.

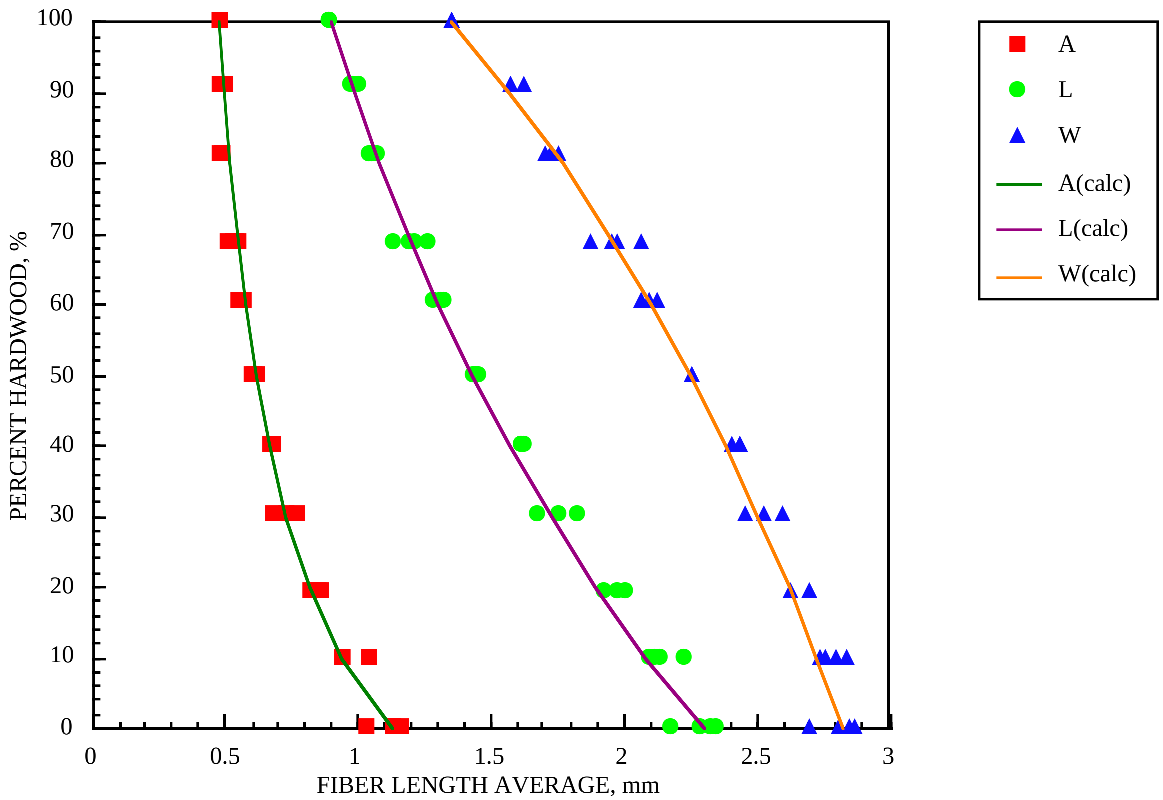
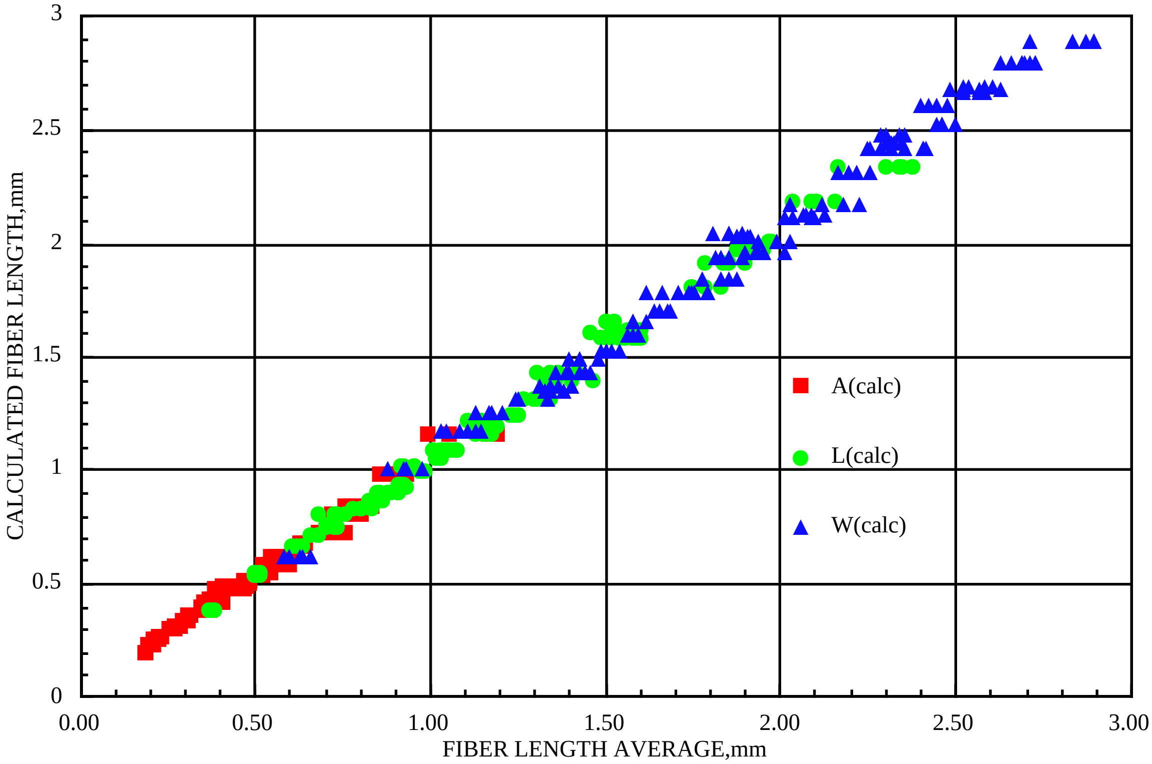


Figure 4. Calculated versus measured fiber length averages for multiple component pulps.




equal to .91%, .65% and .81% for the arithmetic, length-weighted and weight-weighted averages respectively. The good agreement with this data clearly indicates that the absence or presence of lignin is not a factor in the measurement of fiber dimension data.

## WEIGHT-FRACTION CALCULATIONS

Determining the weight fractions of component pulps from a mixed pulp distribution is the real power of equations (13)-(15). For binary or two-component mixtures, solving these equations for the weight fraction is a simple algebraic task and results in a single equation with one unknown. However, three separate equations can be derived from equations (13)-(15):

$$x_1 = \frac{\overline{A}_1 Q_1\left(\overline{A}_2 - \overline{A}_{mix}\right)}{\overline{A}_1 Q_1\left(\overline{A}_2 - \overline{A}_{mix}\right) + \overline{A}_2 Q_2\left(\overline{A}_{mix} - \overline{A}_1\right)} \qquad (20)$$

$$x_1 = \frac{Q_1\left(\overline{L}_2 - \overline{L}_{mix}\right)}{Q_1\left(\overline{L}_2 - \overline{L}_{mix}\right) + Q_2\left(\overline{L}_{mix} - \overline{L}_1\right)}. \qquad (21)$$

$$x_1 = \frac{\overline{L}_2 Q_1\left(\overline{W}_2 - \overline{W}_{mix}\right)}{\overline{L}_2 Q_1\left(\overline{W}_2 - \overline{W}_{mix}\right) + \overline{L}_1 Q_2\left(\overline{W}_{mix} - \overline{W}_1\right)} \qquad (22)$$

Having three equations and only one unknown, an “over-determined” condition exists. Ideally, if there were no experimental error in the data, either of the above equations would work equally well. Applying equations (20)-(22) to the binary data of Table VI and using the data from Table III, it becomes apparent that equation (21) is the best single

Table III Fiber dimension data for component pulps.

| | A, mm | L, mm | W, mm | Q, g/mm |
|---|---|---|---|---|
| HWD | .461 | .893 | 1.350 | .115 |
| SGW | .190 | .378 | .620 | .220 |
| SWD | 1.135 | 2.320 | 2.847 | .208 |
| TMP | .520 | 1.320 | 1.948 | .263 |

Table IV Comparison of weight fraction prediction.

| Equation | Intercept | Slope | $R^2$ |
|---|---|---|---|
| (20) | -.020 | 1.038 | .860 |
| (21) | .016 | .996 | .945 |
| (22) | .080 | .928 | .847 |

equation solution. Shown in Table IV, the results of regressing the predicted weight fraction against the actual weight fraction has an intercept closer to zero, a slope closer to unity and a higher coefficient of determination ($R^2$) for equation (21) than the other two weight fraction equations. This result corresponds to the higher degree of linearity exhibited for the "length-weighted" relationship in Figure 3, and to the fact that the area under the length-weighted distribution is proportional to the weight of the pulp with the incorporation of fiber coarseness (Eq. (8)).

A more general solution for the determination of weight fraction involves linear programming techniques and is not limited to two components. For solutions up to five component pulps the five equations presented in Table V should be used. These equations are simply algebraic re-expressions of equations (13)-(16) to equate them to zero. When there are fewer than five unknowns, then least squares techniques appropriate for over-determined systems should be used. When the number of component pulps exceeds five, the number of equations should be expanded to match the number of unknowns. New equations can be derived by calculating higher moment averages (similar to common practice for polymeric molecular-weight distributions). For instance, six component pulps can be handled by introducing a "Z" fiber length average defined as

$$\overline{Z} = \frac{\sum nl^4}{\sum nl^3}. \tag{23}$$

Which requires a multiple component formulation of

$$\overline{Z}_{mix} = \frac{\sum_j \frac{\overline{Z}_j \overline{W}_j \overline{L}_j x_j}{Q_j}}{\sum_j \frac{\overline{W}_j \overline{L}_j x_j}{Q_j}}, \tag{24}$$

and a simultaneous equation formulation of

$$\sum_j \left( \frac{\overline{W}_j \overline{L}_j x_j}{Q_j} \right) \left( \overline{Z}_{mix} - \overline{Z}_j \right) = 0. \tag{25}$$

In similar fashion, Z+n averages can be calculated for any additional equations.

The least squares approach, described above, was used to estimate the weight fraction of all component pulps in Table VI. In a paired t-test between the actual and the calculated weight fractions, the mean difference was 1.7% with a standard error of .5% for 328 pairs.

Table V Simultaneous equations for weight fraction calculation.

$$\sum_j x_j = 1$$

$$\sum_j \left( \frac{\overline{A}_{mix} - \overline{A}_j}{\overline{A}_j Q_j} \right) x_j = 0$$

$$\sum_j \left( \frac{\overline{L}_{mix} - \overline{L}_j}{Q_j} \right) x_j = 0$$

$$\sum_j \left( \frac{\overline{L}_j x_j}{Q_j} \right) \left( \overline{W}_{mix} - \overline{W}_j \right) = 0$$

$$Q_{mix} \sum_j \frac{x_j}{Q_j} = 1$$

Table VI. Average data from multiple component blending experiment.

| % Pulp Composition | | | | A, mm | | L, mm | | W, mm | | Q, g/m | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SWD | HWD | SGW | TMP | (exp) | (calc) | (exp) | (calc) | (exp) | (calc) | (exp) | (calc) |
| 0 | 0 | 20 | 80 | 0.35 | 0.39 | 1.07 | 1.09 | 1.84 | 1.84 | 0.26 | 0.24 |
| 0 | 0 | 40 | 60 | 0.28 | 0.31 | 0.86 | 0.89 | 1.67 | 1.70 | 0.24 | 0.23 |
| 0 | 0 | 60 | 40 | 0.24 | 0.26 | 0.68 | 0.70 | 1.44 | 1.49 | 0.23 | 0.22 |
| 0 | 0 | 80 | 20 | 0.21 | 0.23 | 0.51 | 0.53 | 1.10 | 1.17 | 0.24 | 0.21 |
| 0 | 20 | 0 | 80 | 0.49 | 0.51 | 1.16 | 1.16 | 1.78 | 1.78 | 0.20 | 0.20 |
| 0 | 20 | 80 | 0 | 0.22 | 0.25 | 0.51 | 0.54 | 0.93 | 1.00 | 0.21 | 0.18 |
| 0 | 25 | 25 | 50 | 0.36 | 0.38 | 0.93 | 0.93 | 1.59 | 1.59 | 0.18 | 0.18 |
| 0 | 25 | 50 | 25 | 0.28 | 0.31 | 0.73 | 0.75 | 1.36 | 1.37 | 0.20 | 0.18 |
| 0 | 40 | 0 | 60 | 0.48 | 0.50 | 1.03 | 1.06 | 1.59 | 1.64 | 0.19 | 0.17 |
| 0 | 40 | 60 | 0 | 0.27 | 0.30 | 0.63 | 0.66 | 1.08 | 1.16 | 0.18 | 0.15 |
| 0 | 50 | 25 | 25 | 0.37 | 0.39 | 0.86 | 0.86 | 1.43 | 1.43 | 0.16 | 0.15 |
| 0 | 60 | 0 | 40 | 0.47 | 0.49 | 0.98 | 0.99 | 1.52 | 1.53 | 0.16 | 0.14 |
| 0 | 60 | 40 | 0 | 0.31 | 0.35 | 0.72 | 0.76 | 1.18 | 1.25 | 0.16 | 0.14 |
| 0 | 80 | 0 | 20 | 0.46 | 0.48 | 0.92 | 0.94 | 1.41 | 1.43 | 0.14 | 0.13 |
| 0 | 80 | 20 | 0 | 0.38 | 0.41 | 0.81 | 0.83 | 1.28 | 1.31 | 0.13 | 0.12 |
| 20 | 0 | 0 | 80 | 0.58 | 0.62 | 1.52 | 1.58 | 2.21 | 2.31 | 0.25 | 0.24 |
| 20 | 0 | 80 | 0 | 0.22 | 0.25 | 0.73 | 0.80 | 1.85 | 2.04 | 0.27 | 0.20 |
| 20 | 80 | 0 | 0 | 0.48 | 0.52 | 1.04 | 1.08 | 1.69 | 1.78 | 0.13 | 0.12 |
| 25 | 25 | 0 | 50 | 0.56 | 0.58 | 1.39 | 1.39 | 2.14 | 2.17 | 0.19 | 0.18 |
| 25 | 25 | 25 | 25 | 0.39 | 0.42 | 1.17 | 1.19 | 2.06 | 2.11 | 0.18 | 0.17 |
| 25 | 25 | 50 | 0 | 0.30 | 0.34 | 0.94 | 1.01 | 1.90 | 2.03 | 0.17 | 0.17 |
| 25 | 50 | 0 | 25 | 0.54 | 0.55 | 1.25 | 1.24 | 1.99 | 2.01 | 0.14 | 0.15 |
| 25 | 50 | 25 | 0 | 0.39 | 0.43 | 1.04 | 1.08 | 1.85 | 1.94 | 0.14 | 0.14 |
| 40 | 0 | 0 | 60 | 0.71 | 0.72 | 1.78 | 1.80 | 2.47 | 2.52 | 0.22 | 0.22 |
| 40 | 0 | 60 | 0 | 0.29 | 0.31 | 1.14 | 1.21 | 2.33 | 2.47 | 0.22 | 0.20 |
| 40 | 60 | 0 | 0 | 0.56 | 0.57 | 1.31 | 1.31 | 2.09 | 2.13 | 0.14 | 0.13 |
| 50 | 0 | 25 | 25 | 0.42 | 0.48 | 1.52 | 1.64 | 2.44 | 2.60 | 0.22 | 0.20 |
| 50 | 25 | 0 | 25 | 0.64 | 0.67 | 1.59 | 1.62 | 2.33 | 2.43 | 0.17 | 0.17 |
| 50 | 25 | 25 | 0 | 0.43 | 0.47 | 1.36 | 1.42 | 2.29 | 2.41 | 0.19 | 0.16 |
| 60 | 0 | 0 | 40 | 0.80 | 0.84 | 1.94 | 2.00 | 2.57 | 2.68 | 0.21 | 0.21 |
| 60 | 0 | 40 | 0 | 0.38 | 0.42 | 1.53 | 1.60 | 2.56 | 2.68 | 0.19 | 0.19 |
| 60 | 40 | 0 | 0 | 0.63 | 0.66 | 1.59 | 1.57 | 2.39 | 2.42 | 0.15 | 0.15 |
| 80 | 0 | 0 | 20 | 0.90 | 0.98 | 2.10 | 2.17 | 2.68 | 2.79 | 0.20 | 0.20 |
| 80 | 0 | 20 | 0 | 0.58 | 0.62 | 1.92 | 1.97 | 2.70 | 2.80 | 0.16 | 0.19 |
| 80 | 20 | 0 | 0 | 0.77 | 0.81 | 1.85 | 1.90 | 2.57 | 2.66 | 0.15 | 0.16 |

## CONCLUSIONS

Modern optical fiber-length analysis is a very powerful analytical technique. The data obtained from these new instruments is more accurate and detailed than previous techniques based on classification. The use of a single fiber-length average (particularly the weight-weighted average) to analyze fiber-length distributions is rooted in the inherent limitations of classification and should be discontinued with these instruments. The use of all three common fiber-length averages (arithmetic, length-weight and weight-weight) is preferable because the shape or dispersion of a fiber-length distribution is described in more detail.

The area of a length-weighted distribution (nl versus l) is proportional to the weight of the pulp through the incorporation of fiber coarseness. The shape of this distribution is useful in analyzing the proper proportions of fines versus long-fiber content. It also permits the calculation of weight-fraction of component pulps in blended pulps.

The calculation of component pulp weight-fractions requires the calculation of at least the three common fiber-length averages plus coarseness for blends of up to five components. Additional components require the calculation of higher averages that have not been used in pulp analysis but are common in the molecular weight analysis of polymers, notably the Z, Z+1, etc..

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