

PROMPTING TEACHER KNOWLEDGE DEVELOPMENT USING DYNAMIC GEOMETRY SOFTWARE

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Teachers' mathematical knowledge for teaching (MKT) has been found to impact teacher change (Hill, Rowan, & Ball, 2004). This study examines teachers' development of MKT as they implement dynamic geometry software (DGS) with a coach. We relied on Laborde's (2001) account of expertise in teaching using DGS as an analytical tool to categorize teachers' ways of using DGS to teach geometric reasoning. We found a relation between teachers' expressions of MKT and their ways of using DGS, based on qualitative evidence.

Introduction and Theoretical Framework

Teacher knowledge has taken center stage as researchers seek to improve mathematics education in the United States. However, content-specific professional development has not brought about significant teacher change. Alternatively, several studies have surfaced that point to specialized teacher knowledge as a key to effective reform. The seminal study relating teacher knowledge to student achievement found that *mathematical knowledge for teaching* (MKT) was a key to predicting student gains in first and third grade (Hill, Rowan, & Ball, 2004). We explore ways of improving one aspect of MKT, *geometric reasoning*, as teachers participate in collaborative professional development emphasizing the use of dynamic geometry software (DGS) in their classrooms.

Components of Mathematical Knowledge for Teaching (MKT)

Subject Matter Knowledge	Pedagogical Content Knowledge
Common Content Knowledge (CCK)	Knowledge of Content and Students (KCS)
Specialized Content Knowledge (SCK)	Knowledge of Content and Teaching (KCT)
Knowledge at the Mathematical Horizon	Knowledge of Curriculum

Table 1. Components of MKT

Ball, Bass, Goffney, and Sleep (2006) defined mathematical knowledge for teaching as the “mathematical knowledge, skills, [and] habits of mind that are entailed by the work of teaching” (Slide 5). More specifically, Ball and her colleagues categorize mathematical knowledge for teaching MKT in six parts. Common content knowledge is basic, lay-person knowledge of the mathematical content. Specialized content knowledge (SCK) is the way the mathematics arises in classrooms, such as for building representations. Knowledge of content and students (KCS), is knowing how students think about mathematics. Knowledge of content and teaching (KCT) involves knowing the most effective examples or teaching sequences. We understand Shulman’s (1987) definition of pedagogical content knowledge to be a marriage of knowledge of content and students (KCS) with knowledge of content and teaching (KCT). Knowledge of curriculum and knowledge at the mathematical horizon are the final components of mathematical knowledge for teaching. In this study, we focused on specialized content knowledge (SCK), knowledge of content and students (KCS), and knowledge of content and teaching (KCT). We believe teachers

engage students in substantive mathematics as they use DGS to relate and contrast variant and invariant elements in constructed objects and attend to special cases and boundary cases (Arshavsky & Goldenberg, 2005; Laborde, 2001; Presmeg, Barrett, & McCrone, 2007).

Because of the demonstrated link between mathematical knowledge for teaching and student achievement, this study sought, through microgenetic analysis, to unpack the psychological processes of how teachers develop MKT. The teachers in our study were engaged in professional development through an NSF-funded Graduate Teaching Fellows in K-12 Education (GK-12) project (Moore, 2003). GK-12 places graduate mathematics and science fellows in classrooms as *collaborative coaches* who design and deliver standards-based instruction with classroom teachers. Thus, the GK-12 model of professional development allows the transfer from theory to practice, a task which Bazzini and Morselli (2006) have identified as worthwhile, yet complex.

Research Questions

Because appropriate use of dynamic geometry software affords teachers new opportunities for discourse with their students we posed the following research question and sub-questions:

In what ways do teachers develop mathematical knowledge for teaching as they engage with a collaborative coach to prepare and implement inquiry-based lessons within a dynamic geometry computer environment? In particular in the DGS environment:

1. In what ways do teachers develop common content knowledge (CCK) and specialized content knowledge (SCK)?
2. In what ways do teachers develop knowledge of content and students (KCS) and knowledge of content and teaching (KCT)?

Methods and Analysis

Four 5th-8th grade teachers and three GK-12 fellows participated as case studies in this multi-tiered teacher development experiment (Presmeg & Barrett, 2003; Lesh & Kelly, 2000). However, for purposes of this report, we narrowed our focus to the mathematical knowledge for teaching development of one case study teacher. Mrs. Gerber collaborated with a fellow to teach 22 Sketchpad-based lessons in her 6-8th grade classes over the course of two years. To collect data, we videotaped all but three of the 22 classes. In addition to videotapes, the teacher participated in interviews before and after each lesson and before and after the unit of Sketchpad-based lessons.

To triangulate the data, the fellows wrote pre and post reflections of the lessons. Moreover, to establish trustworthiness, the fellow from the second year (the third author of this report) engaged in respondent validation. The first author of this report acted as a participant-observer. Data was analyzed for development in mathematical knowledge for teaching with regard to common content knowledge, specialized content knowledge, knowledge of content and students, and knowledge of content and teaching. Inter-rater reliability of 81% was established among codes for identical portions of at least three transcripts per fellow, after which the first author coded the remaining transcripts.

To identify development of specialized content knowledge, data was further analyzed with respect to the levels of DGS use ascribed to novice versus expert teachers by Laborde (2001). Level one involved creating and measuring figures. Level 2 added dragging to amplify and explore geometric properties. Level three tasks allowed for dynamic solution strategies, although

they continued to have paper and pencil counterparts. Level four tasks derived their meaning from and depended on dynamic geometry. The first and second author coded lessons for Laborde levels independently. We then reconciled differences by considering further details of the lessons and by reading and discussing observer field notes and fellow's written reflections.

Results and Discussion

To illustrate Mrs. Gerber's development of MKT, we discuss a classroom episode from each year. In the first episode, the students had constructed tessellations based on translating modified parallelograms by a marked vector. During the post reflection, the teacher and GK-12 fellow (first author) discussed how students had produced a tessellation with a hole in it. As they talked, it became apparent that Mrs. Gerber believed that the concept of vector depended on *position*, a misconception similar to that of a student described by Hollebrands (2007). In response, the fellow conveyed to her that vectors involve direction and distance, but not position. Thus, reflection, along with the capacity of Sketchpad to amplify what was variant (position) and what was not (vector), revealed the teacher's misconception.

Mrs. Gerber developed three components of mathematical knowledge for teaching in this episode. First, she developed specialized content knowledge (**SCK**) as she observed the fellow teaching students about vectors on Sketchpad. She commented, "I wasn't thinking of the vector showing the direction. And you [the fellow] made the comment, you said that that vector's going to show direction and distance. I thought, 'Yeah, that is what it's doing. It's moving it over.'" Her observation of the fellow in a practice-based setting enhanced her learning above the common content knowledge that she may have learned previously in a lecture course. Secondly, she developed specialized content knowledge (**SCK**) as she reflected with the fellow about a student production using DGS. The fellow identified her misconception and corrected her using the context of student work. Thirdly, she developed knowledge of content and students (**KCS**) as she and the fellow considered what the students had done wrong. Finally, the teacher developed knowledge of content and teaching (**KCT**) as she gained confidence to teach geometry using Sketchpad by watching the fellow teach and by answering student questions with the fellow present for support. During the post reflection, the fellow asked, "Do you feel comfortable enough to [use the DGS] with 6th grade [without my assistance]?" She responded, "I think I do if I can finish with 8th grade and yeah, because by 7th grade I felt better. I knew what I was doing." She had observed and co-taught the same tessellation lesson to first 8th and then 7th grade. Later in the year, she taught the lesson alone to her 6th grade class, providing evidence of growth.

In the second year a series of lessons using DGS was implemented to teach quadrilateral properties. To conclude the unit, the teacher and fellow collaboratively developed a lesson to assess whether students had learned how to classify quadrilaterals according to their properties. They decided to provide students with an unbreakable kite on Sketchpad, and assign them the task of identifying its properties and placing it within an existing taxonomy of quadrilaterals (See Figure 1). During the pre-lesson planning meeting, the fellow explained his idea for the task, which was beyond what the textbook presented. They discussed including *always*, *sometimes*, and *never* relationships within the arrows of the taxonomy which moved from general to specific figures (Battista, 1998). Together, Mrs. Gerber and the fellow decided that the model held true for the *always* propositions as one might travel upwards through the taxonomy, and the *sometimes* propositions held true as one moves down through the taxonomy. For example, a square is *always* a rhombus because it is located below the rhombus and connected by a down

arrow in the taxonomy. As one moves upwards, from square to rhombus, one finds that a given square is always a rhombus. Alternatively, a rhombus is *sometimes* a square (as one would find by moving downwards along the arrow). They realized that propositions labeled *never* would not fit this taxonomy.

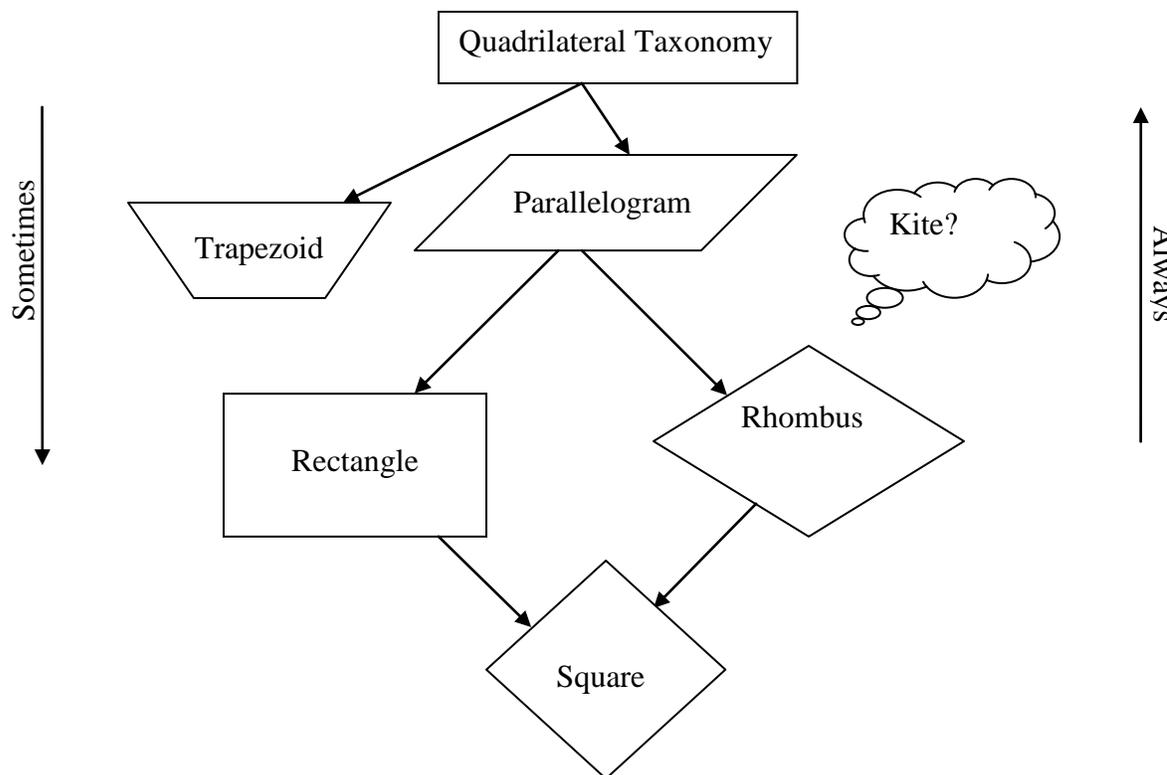


Figure 1. Quadrilateral Taxonomy

During the actual lesson, the teacher and a pair of students dragged the kite into the shape of a rhombus. Together they decided that a kite should have an arrow from it, downwards to a rhombus, signifying that a kite is sometimes a rhombus. This would match their experience in moving the kite around and forming it into a rhombus. Five months later in her final interview, Mrs. Gerber was not immediately certain that there should be an arrow from the kite to the rhombus. After reasoning through the properties of the figures in the taxonomy, she deduced that there should be an arrow showing that a kite is sometimes a rhombus. However, she indicated that she would like to check it in her notes to be sure, as can be seen in the following quote from her final interview.

Mrs. Gerber: Well and he [the fellow] did them, we did them in school too, but then we were trying to make sure that when we made this diagram that the kids could see where it was coming from because we were going with sometimes and always.

AK: OK

Mrs. Gerber: And so from the quadrilateral, uh, a quadrilateral is sometimes a parallelogram. Then, a parallelogram is sometimes a rhombus, or it's sometimes a rectangle.

AK: OK

Mrs. Gerber: OK. [Pause.] And then a rectangle or a rhombus is sometimes a square.

AK: So you had arrows going to the square.

Mrs. Gerber: Mhm, to the square. And the square was down at the end. Now, the trapezoid was up here. A quadrilateral is sometimes a trapezoid. But we put it over to the side because a trapezoid does not fit with the parallelogram. The properties don't. Or the rectangle or rhombus or square. And then we also had a kite. And kite came over here, and a quadrilateral is sometimes a kite. Now, you know what? I think we had an arrow from the kite to the rhombus. [long pause] I'd have to go back and look at the papers. But I think we did relate it down here that if the kite had four congruent sides, it could be a rhombus because your opposite angles, not opposite angles, well the ones opposite each other yeah, they would be could be congruent. I mean you could make a kite that way, and it would be OK. And so I think we had, I'm going to do this. [She drew a dashed arrow from the kite to the rhombus.]

Mrs. Gerber was finding that she did not have to recall every fact of geometry, nor look it up in a reference book. Instead, she was comfortable recalling this reasoning process on her own, even five months later. Another issue from the lesson arose when the fellow asked the class if a square is a kite. Students struggled with the question. Later, during a post reflection, the teacher and fellow discussed the reason for the students' difficulty and related it back to the property that a kite has *at least one* bisected diagonal. Students may have been thinking of a kite as having *exactly one* bisecting diagonal, which would have excluded the square (cf. Scher, 2005).

Mrs. Gerber again developed three elements of mathematical knowledge for teaching during this second episode. First, she and the fellow both developed specialized content knowledge (**SCK**) as they discussed the labeling of *always*, *sometimes* and *never* within the taxonomy. Mrs. Gerber developed knowledge of content and teaching (**KCT**) as she and the fellow raised the Laborde (2001) level of the lesson to level two from how she typically taught at level one or from a traditional text book without any use of technological tools. Using DGS *to study variation* contributed to the teacher's development of specialized content knowledge (**SCK**) as she and her students dragged the kite into a rhombus and discussed the relation of rhombus and kite. Her procedural knowledge of *properties* had developed into knowledge of *relationships* among properties as she engaged in pseudo-deductive reasoning in her final interview (Hollebrands, 2007). Moreover, her confidence to conjecture with students on Sketchpad indicates she was developing profound understanding of fundamental mathematics (Ma, 1999) and **SCK**. DGS further allowed Mrs. Gerber to hear her students and see their actions on DGS simultaneously, which led to knowledge of content and students (**KCS**). For example, as a student investigated the properties of a kite on sketchpad, the following discourse ensued:

Mrs. Gerber: Now you said opposite angles are congruent. So are you saying this one and this one?

Student: Yeah.

Mrs. Gerber: What about these two?

Student: No! [student modifies the property that he had typed.]

By questioning the student in relation to his sketch, Mrs. Gerber was able to figure out that a student had discovered one set of congruent opposite angles in a kite, but had neglected to investigate the other set. She learned that it took students *time* to reason through properties of

figures, and she was surprised that some of her students who did not typically excel on paper and pencil tasks flourished in an open-ended DGS environment. This KCS result builds on Hollebrands' (2007) account of such an environment "by discussing or referring to a common shared screen, the teacher and student can communicate mathematical ideas through the use of visual images" (p. 168). Finally, as with the first episode, the post reflection allowed for the fellow and teacher to discuss a perturbation from the lesson. It was decided that a quadrilateral having *at least one* perpendicularly bisecting diagonal best defined the kite. Thus, the teacher developed specialized content knowledge (**SCK**) during the post reflection.

Student understanding in the second year was assessed through a 23-item pre/post test written instrument. The 29 7th-grade students were asked to respond with *always*, *sometimes*, and *never* to statements relating various subclasses of quadrilaterals. For example, one question stated, "A parallelogram is a rhombus." Students performed significantly better on the post test than on the pre test ($p=.000$). The mean score for the pretest was 9.4, whereas the mean for the post test was 14.7, with an average 5.3 point increase in scores. Using DGS facilitated student understanding and teacher knowledge.

Conclusions and Implications

The two episodes described here typify the coaching arrangement of this teacher with fellows and their use of a DGS. Lasting change in Mrs. Gerber's mathematical knowledge for teaching, however, can best be seen across the two-year span. In the first year, she focused largely on "teaching Sketchpad," whereas in the second year her focus was better characterized as "teaching geometry using Sketchpad as a tool." For example, learning to use DGS as a tool for showing geometric ideas facilitated her mathematical definitions of those ideas or objects since she found herself explaining to students what particular dragging or constructing actions within the software meant in terms of the geometric objects and relations among them. She learned to attend to student screens to find what aspect of her explanations or definitions they did not yet understand. She could then decide how to clarify her definition, thus developing specialized content knowledge (**SCK**).

Rather than relying solely on DGS lessons from the GK-12 program, she began in the second year to teach and envision her own DGS lessons. Lessons taught with a fellow were on average neatly one Laborde (2001) level above her own teaching, allowing her to develop knowledge of content and students (**KCS**) as she observed her students learning through dynamic processes. As Laborde stated, "a deep and precise knowledge of students' behavior and strategies" in DGS is an essential part of a teacher's knowledge (p. 309). Furthermore, Mrs. Gerber's lessons in the second year were designed to introduce concepts using DGS rather than merely working to reinforce them, a shift indicative of movement from a novice user of DGS toward expertise.

Thus, the practice-based, collaborative professional development environment established by the GK-12 program, along with the visualization afforded by dynamic geometry facilitated her development of geometric reasoning and mathematical knowledge for teaching. With respect to the first research question, Mrs. Gerber developed specialized content knowledge (SCK) to a much greater degree than common content knowledge (CCK). First, the invariant features of dynamic geometry facilitated richer mathematical discourse between the teacher and her students. Secondly, she developed SCK through exploration with her students in a DGS environment. Third, she gained mathematical knowledge by observing the fellow teach and by reflecting with the fellow about mathematical topics arising from her practice.

For the second research question, Mrs. Gerber developed knowledge of content and students (KCS) as she discussed student work and misconceptions with the fellow. Furthermore, she developed KCS as she observed and listened to students as they dragged figures on DGS. Finally, Mrs. Gerber developed knowledge of content and teaching (KCT) by watching the fellow teach at a Laborde (2001) level higher than her own. By helping students with a fellow present for support she gained confidence to implement “goal free” instruction, which Manizade (2006) identified as an element of pedagogical content knowledge of geometry and measurement for middle school. Thus, Mrs. Gerber demonstrated development and growth in mathematical knowledge for teaching throughout her two years of implementing DGS with the assistance of a collaborative coach. This in turn translated to deeper geometric reasoning for her students, substantiating Hill, Rowan, and Ball’s (2004) finding that increased MKT of teachers supports student learning. Further study is needed to elaborate on coaching strategies and to link such DGS coaching to student achievement. This study was supported by National Science Foundation Grant # DGE-0338188.

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